

## MATH 102.5 (Term 142)

Quiz 4 (Sects. 8.2, 8.3 &amp; 8.4)

Duration: 30mn

Name:

ID number:

1.) (4pts) Evaluate the integral  $I = \int \frac{1}{\sqrt{4x^2 - 9}} dx, \quad x > 3/2.$ 2.) (6pts) Evaluate the integral  $J = \int \frac{1}{(x^2 - 3x + 2)(x+1)} dx.$ 3.) Write the function  $f(x) = \frac{2x^4 + 1}{x^3(2x^2 + x + 1)^2}$  in partial fraction (Do not evaluate the constants).

$$\begin{aligned} 1.) \quad x &= \frac{3}{2} \sec \theta, \quad 0 < \theta < \frac{\pi}{2} \\ dx &= \frac{3}{2} \sec \theta \tan \theta d\theta \\ 4x^2 - 9 &= 9 \tan^2 \theta, \quad \tan \theta > 0 \\ \Rightarrow I &= \int \frac{\frac{3}{2} \sec \theta \tan \theta}{3 \tan \theta} d\theta \\ &= \frac{1}{2} \int \sec \theta d\theta \quad \begin{array}{c} \text{triangle} \\ \theta \quad \sqrt{4x^2 - 9} \\ \hline 3 \end{array} \\ &= \frac{1}{2} \ln |\sec \theta + \tan \theta| + C \end{aligned}$$

$$\begin{aligned} x &= \frac{3}{2} \sec \theta \Rightarrow \sec \theta = \frac{2}{3} x \\ 1 + \tan^2 \theta &= \sec^2 \theta \Rightarrow \tan \theta = \frac{1}{3} \sqrt{4x^2 - 9} \end{aligned}$$

$$I = \frac{1}{2} \ln \left| \frac{2}{3} x + \frac{1}{3} \sqrt{4x^2 - 9} \right| + C$$

$$\begin{aligned} 2.) \quad x^2 - 3x + 2 &= (x-1)(x-2) \\ \frac{1}{(x^2 - 3x + 2)(x+1)} &= \frac{a}{x-1} + \frac{b}{x-2} + \frac{c}{x+1} \end{aligned}$$

$$\begin{aligned} a &= \left. \frac{1}{(x-1)(x+1)} \right|_{x=1} = -\frac{1}{2} \\ b &= \left. \frac{1}{(x-1)(x+1)} \right|_{x=2} = \frac{1}{3} \\ c &= \left. \frac{1}{(x-1)(x+1)} \right|_{x=-1} = \frac{1}{6} \\ J &= \int \left[ -\frac{1}{2(x-1)} + \frac{1}{3(x-2)} + \frac{1}{6(x+1)} \right] dx \\ &= -\frac{1}{2} \ln|x-1| + \frac{1}{3} \ln|x-2| + \frac{1}{6} \ln|x+1| + C \\ 3.) \quad f(x) &= \frac{a}{x} + \frac{b}{x^2} + \frac{c}{x^3} + \frac{dx+e}{2x^2+x+1} \\ &\quad + \frac{fx+g}{(2x^2+x+1)^2} \end{aligned}$$

## MATH 102.29 (Term 142)

Quiz 4 (Sects. 8.1, 8.3 &amp; 8.4)

Duration: 30mn

Name:

ID number:

1.) (4pts) Evaluate the integral  $I = \int \frac{1}{x^2\sqrt{4x^2+9}} dx$ .2.) (4pts) Evaluate the integral  $J = \int \frac{1}{(x^2+3x+2)(x-1)} dx$ .3.) (2pts) Write the function  $f(x) = \frac{x^5-1}{x^2(3x^2+x+1)^3}$  in partial fraction (Do not evaluate the constants).

$$1.) \quad x = \frac{3}{2} \tan \theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$4x^2 + 9 = 9(1 + \tan^2 \theta) = 9 \sec^2 \theta$$

$$dx = \frac{3}{2} \sec^2 \theta d\theta, \quad \sec \theta > 0$$

$$\Rightarrow I = \int \frac{\frac{3}{2} \sec^2 \theta d\theta}{27/4 \sec \theta \tan^2 \theta}$$

$$= \frac{2}{9} \int \frac{\sec \theta}{\tan^2 \theta} d\theta$$

$$= \frac{2}{9} \int \frac{\cos \theta}{\sin^2 \theta} d\theta$$

$$= -\frac{2}{9} \frac{1}{\sin \theta} + C$$

$$x = \frac{3}{2} \tan \theta \Rightarrow \sin \theta = \frac{2}{3} x \cos \theta$$

$$1 + \tan^2 \theta = \sec^2 \theta \Rightarrow \cos \theta = \frac{3}{\sqrt{4x^2+9}}$$

$$\Rightarrow \sin \theta = \frac{2x}{\sqrt{4x^2+9}}$$

$$I = -\frac{\sqrt{4x^2+9}}{9x} + C$$

$$2.) \quad (x^2+3x+2) \neq (x+1)(x+2)$$

$$\frac{1}{x^2+3x+2} = \frac{a}{x+1} + \frac{b}{x+2} + \frac{c}{x-1}$$

$$\Rightarrow a = \frac{1}{(x+2)(x-1)} \Big|_{x=-1} = -\frac{1}{2}$$

$$b = \frac{1}{(x-1)(x+1)} \Big|_{x=-2} = \frac{1}{3}$$

$$c = \frac{1}{(x+1)(x+2)} \Big|_{x=1} = \frac{1}{6}$$

$$J = \int \left[ -\frac{1}{2} \frac{1}{x+1} + \frac{1}{3} \frac{1}{x+2} + \frac{1}{6} \frac{1}{x-1} \right] dx$$

$$= -\frac{1}{2} \ln|x+1| + \frac{1}{3} \ln|x+2| + \frac{1}{6} \ln|x-1| + C$$

$$3.) \quad f(x) = \frac{a}{x} + \frac{b}{x^2} + \frac{ex+d}{3x^2+x+1} + \frac{fx+f}{(3x^2+x+1)^2}$$

$$+ \frac{gx+h}{(3x^2+x+1)^3}$$