MATH 102.5 (Term 142)

Quiz 4 (Sects. 8.2, 8.3 & 8.4) Duration: 30mn

Name: ID number:

1.)(4pts)Evaluate the integral $I = \int \frac{1}{\sqrt{4x^2-9}}dx, \quad x > 3/2.$

2.)(6pts)Evaluate the integral $J = \int \frac{1}{(x^2-3x+2)(x+1)}dx.$

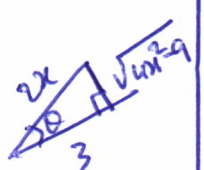
3.)Write the function $f(x) = \frac{2x^4+1}{x^3(2x^2+x+1)^2}$ in partial fraction (Do not evaluate the constants).

1.) $x = \frac{3}{2}\sec\theta$, $0 < \theta < \frac{\pi}{2}$

$dx = \frac{3}{2}\sec\theta\tan\theta\, d\theta$

$4x^2-9 = 9\tan^2\theta$, $\tan\theta > 0$

$\Rightarrow I = \int \frac{\frac{3}{2}\sec\theta\tan\theta}{3\tan\theta}\, d\theta$



$= \frac{1}{2}\int \sec\theta\, d\theta$

$= \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$

$x = \frac{3}{2}\sec\theta \Rightarrow \sec\theta = \frac{2}{3}x$

$1+\tan^2\theta = \sec^2\theta \Rightarrow \tan\theta = \frac{1}{3}\sqrt{4x^2-9}$

$I = \frac{1}{2}\ln\left|\frac{2}{3}x + \frac{1}{3}\sqrt{4x^2-9}\right| + C$

2.) $x^2-3x+2 = (x-1)(x-2)$

$\frac{1}{(x^2-3x+2)(x+1)} = \frac{a}{x-1} + \frac{b}{x-2} + \frac{c}{x+1}$

$a = \left.\frac{1}{(x-2)(x+1)}\right|_{x=1} = -\frac{1}{2}$

$b = \left.\frac{1}{(x-1)(x+1)}\right|_{x=2} = \frac{1}{3}$

$c = \left.\frac{1}{(x-1)(x-2)}\right|_{x=-1} = \frac{1}{6}$

$J = \int\left[\frac{-\frac{1}{2}}{x-1} + \frac{\frac{1}{3}}{x-2} + \frac{\frac{1}{6}}{x+1}\right]dx$

$= -\frac{1}{2}\ln|x-1| + \frac{1}{3}\ln|x-2| + \frac{1}{6}\ln|x+1| + C$

3.) $f(x) = \frac{a}{x} + \frac{b}{x^2} + \frac{c}{x^3} + \frac{dx+e}{2x^2+x+1}$

$+ \frac{fx+g}{(2x^2+x+1)^2}$

MATH 102.29 (Term 142)

Quiz 4 (Sects. 8.2, 8.3 & 8.4) Duration: 30mn

Name: ID number:

1.)(4pts)Evaluate the integral $I = \int \frac{1}{x^2\sqrt{4x^2+9}}dx$.

2.)(4pts)Evaluate the integral $J = \int \frac{1}{(x^2+3x+2)(x-1)}dx$.

3.)(2pts)Write the function $f(x) = \frac{x^5-1}{x^2(3x^2+x+1)^3}$ in partial fraction (Do not evaluate the constants).

1°) $x = \frac{3}{2}\tan\theta$, $-\frac{\pi}{2} < \theta < \pi/2$

$4x^2+9 = 9(1+\tan^2\theta) = 9\sec^2\theta$

$dx = \frac{3}{2}\sec^2\theta\, d\theta$, $\sec\theta > 0$

$\Rightarrow I = \int \frac{\frac{3}{2}\sec^2\theta\, d\theta}{\frac{27}{4}\sec\theta\tan^2\theta}$

$= \frac{2}{9}\int \frac{\sec\theta}{\tan^2\theta}d\theta$

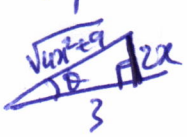
$= \frac{2}{9}\int \frac{\cos\theta}{\sin^2\theta}d\theta$

$= -\frac{2}{9}\frac{1}{\sin\theta} + C$

$x = \frac{3}{2}\tan\theta \Rightarrow \sin\theta = \frac{2}{3}x\cos\theta$

$1+\tan^2\theta = \sec^2\theta \Rightarrow \cos\theta = \frac{3}{\sqrt{4x^2+9}}$

$\Rightarrow \sin\theta = \frac{2x}{\sqrt{4x^2+9}}$



$I = -\frac{\sqrt{4x^2+9}}{9x} + C$

2°) $(x^2+3x+2) = (x+1)(x+2)$

$\frac{1}{x^2+3x+2} = \frac{a}{x+1} + \frac{b}{x+2} + \frac{c}{x-1}$

$\Rightarrow a = \frac{1}{(x+2)(x-1)}\Big|_{x=-1} = -\frac{1}{2}$

$b = \frac{1}{(x-1)(x+1)}\Big|_{x=-2} = \frac{1}{3}$

$c = \frac{1}{(x+1)(x+2)}\Big|_{x=1} = \frac{1}{6}$

$J = \int \left[\frac{-\frac{1}{2}}{x+1} + \frac{\frac{1}{3}}{x+2} + \frac{\frac{1}{6}}{x-1}\right]dx$

$= -\frac{1}{2}\ln|x+1| + \frac{1}{3}\ln|x+2| + \frac{1}{6}\ln|x-1| + C$

3°) $f(x) = \frac{a}{x} + \frac{b}{x^2} + \frac{ex+d}{3x^2+x+1} + \frac{ex+f}{(3x^2+x+1)^2}$

$+ \frac{gx+h}{(3x^2+x+1)^3}$