MATH 102.5 (Term 142)

Quiz 3 (Sects. 7.1, 7.3 & 8.1) Duration: 30mn

Name: **ID number:**

1.)(3pts)If $\coth x = -\frac{4}{3}$, with $x < 0$, then find the value of $\sinh 2x$.
2.)(3pts) Find the slope of the tangent line to the curve $y = \ln(\cosh x)$ at $x = \ln 2$.
3.)(4pts)Evaluate $I = \int_1^e x^3 (\ln x)^2 dx$.

1.) $\cosh^2 x - \sinh^2 x = 1$

$\coth^2 x - 1 = \operatorname{csch}^2 x$

thus, $\operatorname{csch}^2 x = \frac{7}{9}$

Since $x<0$, $\sinh x < 0$ and

$\operatorname{csch} x = -\frac{\sqrt{7}}{3}$ and $\sinh x = -\frac{3}{\sqrt{7}}$.

Now, $\cosh^2 x = 1 + \frac{9}{7} = \frac{16}{7}$

$\Rightarrow \cosh x = \frac{4}{\sqrt{7}}$

$\sinh 2x = 2 \cosh x \sinh x$

$= 2\left(-\frac{3}{\sqrt{7}}\right)\left(\frac{4}{\sqrt{7}}\right) = -\frac{24}{7}$

2.) $y = \ln(\cosh x)$

$\frac{dy}{dx} = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

The slope is $\left.\frac{dy}{dx}\right|_{x=\ln 2}$

$\left.\frac{dy}{dx}\right|_{x=\ln 2} = \frac{e^{\ln 2} - e^{-\ln 2}}{e^{\ln 2} + e^{-\ln 2}} = \frac{2 - \frac{1}{2}}{2 + \frac{1}{2}} = \frac{3}{5}$

3) $u = (\ln x)^2 \rightarrow u' = 2\frac{\ln x}{x}$

$v' = x^3 \rightarrow v = \frac{x^4}{4}$

$I = \left[\frac{x^4 (\ln x)^2}{4}\right]_1^e - \frac{1}{2}\int_1^e x^3 \ln x \, dx$

$u = \ln x \rightarrow u' = \frac{1}{x}$

$v' = x^3 \rightarrow v = \frac{x^4}{4}$

$I = \left[\frac{x^4 (\ln x)^2}{4} - \frac{1}{8} x^4 \ln x\right]_1^e + \frac{1}{8}\int_1^e x^3 dx$

$= \left[\frac{x^4 (\ln x)^2}{4} - \frac{x^4 \ln x}{8} + \frac{1}{32} x^4\right]_1^e$

$= \frac{1}{32}\left(-1 + 5e^4\right)$

MATH 102.29 (Term 142)

Quiz 3 (Sects. 7.1, 7.3 & 8.1) Duration: 30mn

Name: ID number:

1.)(3pts)If $\tanh x = -\frac{4}{5}$, **with** $x < 0$, **then find the value of** $\sinh 2x$.

2.)(3pts)Find the slope of the tangent line to the curve $y = (4x^2 - 1)\operatorname{csch}(\ln 2x)$ **at** $x = 1$.

3.)(4pts)Evaluate $I = \int_1^e \frac{1}{x^3}(\ln x)^2 dx$.

1.) $\cosh^2 x - \sinh^2 x = 1$

$1 - \tanh^2 x = \operatorname{sech}^2 x$

Thus, $\operatorname{sech}^2 x = \frac{9}{25} \Rightarrow \operatorname{sech} x = \frac{3}{5}$

$\Rightarrow \cosh x = \frac{5}{3}$.

Now, $\sinh^2 x = \frac{25}{9} - 1 = \frac{16}{9}$

Since $x<0$, $\sinh x < 0$ and

$\sinh x = -\frac{4}{3}$.

Finally, $\sinh 2x = 2 \sinh x \cosh x$

$= 2\left(-\frac{4}{3}\right)\left(\frac{5}{3}\right) = -\frac{40}{9}$

2.) $y = (4x^2-1)\frac{2}{e^{\ln 2x} - e^{-\ln 2x}} = (4x^2-1)\frac{2}{2x - \frac{1}{2x}}$

$= 4x$

The slope is $\left.\frac{dy}{dx}\right|_{x=1}$

$\frac{dy}{dx} = 4 \Rightarrow \left.\frac{dy}{dx}\right|_{x=1} = 4$.

3.) $u = (\ln x)^2 \rightarrow u' = 2\frac{\ln x}{x}$

$v' = \frac{1}{x^3} \rightarrow v = -\frac{1}{2}\frac{1}{x^2}$

$I = \left[-\frac{1}{2}\frac{(\ln x)^2}{x^2}\right]_1^e + \int_1^e \frac{\ln x}{x^3} dx$

$u = \ln x \rightarrow u' = \frac{1}{x}$

$v' = \frac{1}{x^3} \rightarrow v = -\frac{1}{2}\frac{1}{x^2}$

$I = \left[-\frac{1}{2}\frac{(\ln x)^2}{x^2} - \frac{1}{2}\frac{\ln x}{x^2}\right]_1^e + \frac{1}{2}\int_1^e \frac{1}{x^3} dx$

$= \left[-\frac{1}{2}\frac{(\ln x)^2}{x^2} - \frac{1}{2}\frac{\ln x}{x^2} - \frac{1}{4}\frac{1}{x^2}\right]_1^e$

$= \frac{1}{4}\left(1 - \frac{5}{e^2}\right)$