MATH 102.5 (Term 142)

Quiz 2 (Sects. 5.6, 6.1)

Duration: 20 mn

Name: ID:

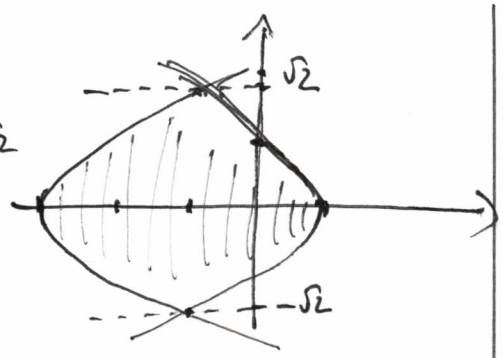
1.) (5 pts) Find the area of the region enclosed by the parabolas $y^2 = x+3$ and $y^2 = -x+1$.

2.) (5 pts) Find the volume of the solid generated by revolving the region enclosed by the triangle with vertices $(-1,0)$, $(-2,2)$ and $(-1,2)$ about the y-axis.

1.) $x+3 = -x+1$

$2x = -2$

$x = -1$, $y = \pm\sqrt{2}$



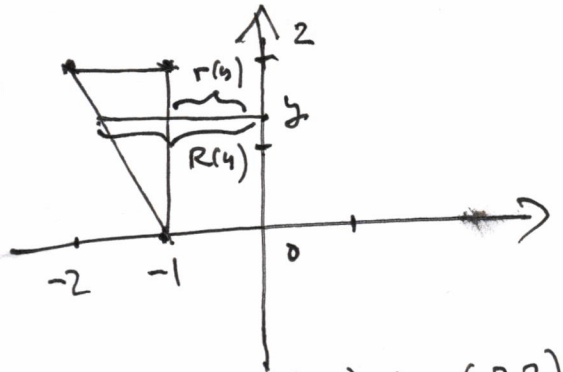
$$\text{Area} = \int_{-\sqrt{2}}^{\sqrt{2}} [-y^2+1-(y^2-3)]\,dy$$

$$= 2\int_0^{\sqrt{2}} (-2y^2+4)\,dy$$

$$= -4\left[\frac{y^3}{3} - 2y\right]_0^{\sqrt{2}}$$

$$= \frac{16\sqrt{2}}{3}$$

2.)



The segment $(-1,0)$ to $(-2,2)$ has equation: $y = -2x-2$

For every $y \in [0,2]$,

$r(y) = 1$, $R(y) = \frac{1}{2}(y+2)$

$$V = \int_0^2 \pi\left[R(y)^2 - r(y)^2\right] dy$$

$$= \pi\int_0^2 \left[\frac{1}{4}(y+2)^2 - 1\right] dy$$

$$= \frac{\pi}{4}\int_0^2 (y^2+2y)\,dy$$

$$= \frac{\pi}{4}\left[\frac{y^3}{3} + y^2\right]_0^2$$

$$= \frac{5}{3}\pi$$

MATH 102.29 (Term 142)

Quiz 2 (Sects. 5.6, 6.1) Duration: 20 mn

Name: ID:

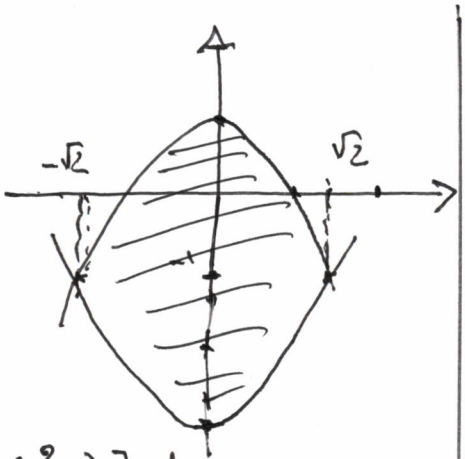
1.) (5pts) Evaluate the area of the region enclosed by the parabolas $x^2 = y+3$ and $x^2 = -y+1$.

2.) (5 pts) The base of a solid is bounded by the curves $y=\sqrt{x}$, $y=0$ and $x=1$. If the cross-section perpendicular to the x-axis are circles, then find the volume of the solid.

1.) $y+3 = -y+1$

$2y = -2$

$y=-1$, $x=\pm\sqrt{2}$



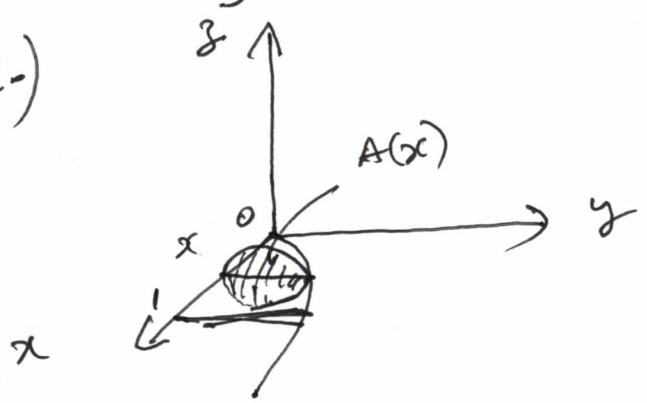
$$\text{Area} = \int_{-\sqrt{2}}^{\sqrt{2}} \left[-x^2+1-(x^2-3)\right] dx$$

$$= 2\int_0^{\sqrt{2}} (-2x^2+4)\, dx$$

$$= -4\left[\frac{x^3}{3} - 2x\right]_0^{\sqrt{2}}$$

$$= \frac{16}{3}\sqrt{2}$$

2.)



For every $x \in [0,1]$,

$A(x)$ is a disk of radius

$r(x) = \frac{\sqrt{x}}{2}$

$$V = \int_0^1 A(x)\, dx, \quad A(x) = \pi r(x)^2 = \pi\frac{x}{4}$$

$$\Rightarrow V = \int_0^1 \pi\frac{x}{4}\, dx$$

$$= \frac{\pi}{4}\left[\frac{x^2}{2}\right]_0^1$$

$$= \frac{\pi}{8}$$