1. If $f(t) = \begin{cases} 3 & \text{if} \quad t \le 0 \\ t^2 - t + 3 & \text{if} \quad 0 < t < 3 \\ 3t & \text{if} \quad t \ge 3 \end{cases}$

then $\int_{-3}^{3} f(t)\,dt = \int_{-3}^{0} 3\,dt + \int_{0}^{3} (t^2 - t + 3)\,dt$

(a) $\frac{45}{2}$ $\quad = 3t\Big]_{-3}^{0} + \Big[\frac{1}{3}t^3 - \frac{1}{2}t^2 + 3t\Big]_0^3$

(b) 0 $\quad = 0 - 3(-3) + 9 - \frac{9}{2} + 9 - 0$

(c) $\frac{27}{2}$ $\quad = 27 - 9/2 = \frac{45}{2}$.

(d) $\frac{21}{2}$

(e) 3

2. The **average value** of $f(x) = \tan x$ on $\left[0, \frac{\pi}{4}\right]$ is

$$f_{av} = \frac{1}{\frac{\pi}{4} - 0} \int_0^{\pi/4} \tan x\,dx$$

(a) $\frac{2}{\pi} \ln 2$

$$= \frac{4}{\pi}\Big[-\ln|\cos x|\Big]_0^{\pi/4}$$

(b) $\frac{4}{\pi}$

$$= \frac{4}{\pi}\left(-\ln\left(\frac{1}{\sqrt{2}}\right) + \ln(1)\right)$$

(c) $\frac{\pi}{4}$

$$= \frac{4}{\pi}\left(-\cancel{\ln(1)}^{\,0} + \ln\sqrt{2} + 0\right)$$

(d) $\frac{3}{\pi}$

(e) $\pi - \ln 2$

$$= \frac{4}{\pi}\left(\frac{1}{2}\ln 2\right) = \frac{2}{\pi}\ln 2.$$

3. $\int \frac{x^3}{x+1}\,dx =$ $\int \frac{x^3+1-1}{x+1}\,dx = \int\left(\frac{(x+1)(x^2-x+1)}{x+1} - \frac{1}{x+1}\right)dx$

(a) $\frac{1}{3}x^3 - \frac{1}{2}x^2 + x - \ln|x+1| + C$

$= \int\left(x^2 - x + 1 - \frac{1}{x+1}\right)dx$

(b) $\frac{1}{4}x^4 - \frac{1}{3}x^3 + \frac{1}{2}x^2 - x - \ln|x+1| + C$

$= \frac{1}{3}x^3 - \frac{1}{2}x^2 + x - \ln|x+1| + C.$

(c) $3x^2 - x + 1 + \ln|x+1| + C$

(d) $x^3 + \frac{1}{2}x^2 - x + \ln|x+1| + C$

(e) $-\frac{1}{2}x^2 + x - 2\ln|x+1| + C$

4. If $\frac{x^2+x+1}{x^3-x^2+x-1} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$, then $4(A+B+C) =$

$x^2 + x + 1 = A(x^2+1) + (Bx+C)(x-1)$

(a) 6

$x = 1$: $3 = 2A \Rightarrow \boxed{A = 3/2}$

(b) 8

coef. of x^2: $1 = A + B \Rightarrow \boxed{B = -1/2}$

(c) 0

Constant term: $1 = A - C \Rightarrow \boxed{C = 1/2}$

(d) 4

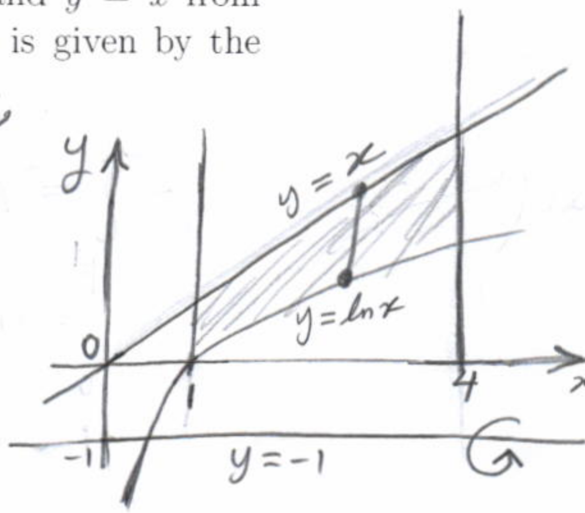
so, $4(A+B+C) = 4\left(\frac{3}{2} - \frac{1}{2} + \frac{1}{2}\right)$

(e) 10

$= 6$

5. The **volume of the solid generated** by revolving the region bounded by the curves $y = \ln x$ and $y = x$ from $x = 1$ to $x = 4$ about the line $y = -1$ is given by the integral.

Using washers,



(a) $\int_1^4 \pi(x^2 - (\ln x)^2 + 2x - 2\ln x)\,dx$

(b) $\int_1^4 \pi(x^2 - (\ln x)^2)\,dx$

(c) $\int_1^4 (x^2 + (\ln x)^2 - 2x\ln x)\,dx$

(d) $\int_1^4 \pi(x^2 + (\ln x)^2 - 2x + 2\ln x + 2)\,dx$

(e) $\int_1^4 \pi(x^2 - (\ln x)^2 + 1)\,dx$

$R(x) = x+1$, $r(x) = \ln x + 1$

$$V = \int_1^4 \pi\left([R(x)]^2 - [r(x)]^2\right)dx$$

$$= \int_1^4 \pi\left((x+1)^2 - (\ln x + 1)^2\right)dx$$

$$= \int_1^4 \pi\left(x^2 + 2x - (\ln x)^2 - 2\ln x\right)dx.$$

6. The **area of the surface** of revolution obtained by revolving the curve $y = \frac{1}{2}x^2$, $0 \le x \le \sqrt{3}$, about the y-axis is

(a) $\frac{14\pi}{3}$

(b) $\frac{16\pi}{3}$

(c) $\frac{21\pi}{2}$

(d) $\frac{24\pi}{5}$

(e) $\frac{12\pi}{5}$

$\frac{dy}{dx} = x$, $1 + \left(\frac{dy}{dx}\right)^2 = 1 + x^2$

$$S = \int_0^{\sqrt{3}} 2\pi x\,ds, \quad ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

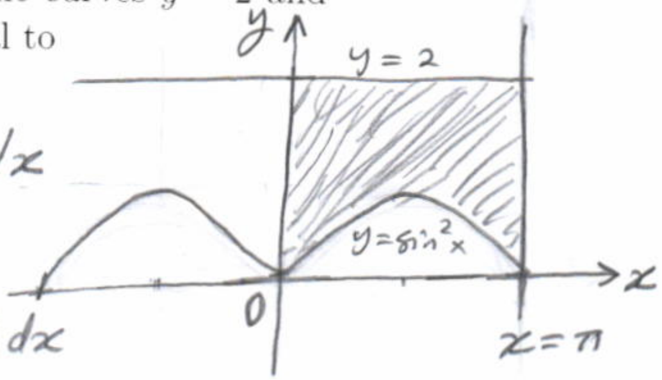
$$= \int_0^{\sqrt{3}} 2\pi x\sqrt{1+x^2}\,dx = 2\pi\int_0^{\sqrt{3}} (1+x^2)^{\frac{1}{2}}(x\,dx)$$

$$= 2\pi\,\frac{1}{2}\,\frac{2}{3}(1+x^2)^{\frac{3}{2}}\Big]_0^{\sqrt{3}}$$

$$= \frac{2\pi}{3}\left((1+3)^{3/2} - 1\right) = \frac{2\pi}{3}(8-1)$$

$$= 14\pi/3.$$

7. The **area** of the region bounded by the curves $y = 2$ and $y = \sin^2 x$ from $x = 0$ to $x = \pi$ is equal to



(a) $\frac{3\pi}{2}$

(b) 1

(c) $\frac{\pi}{2}$

(d) π

(e) 2π

$$A = \int_0^{\pi} (2 - \sin^2 x)\, dx$$

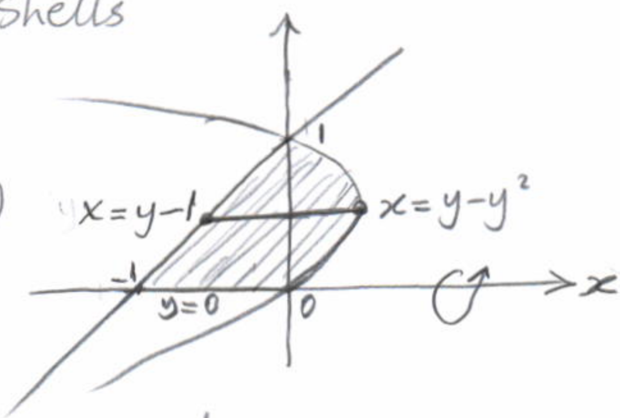
$$= \int_0^{\pi} \left(2 - \frac{1}{2} + \frac{1}{2}\cos 2x\right) dx$$

$$= \frac{3}{2}x + \frac{1}{4}\sin 2x \Big]_0^{\pi}$$

$$= \frac{3\pi}{2} + 0 - 0 - 0 = \frac{3\pi}{2}.$$

8. Let R be the region that lies in the **first and second quadrants** and bounded by the graphs of $x = y - y^2$, $x = y - 1$, and $y = 0$. If R is rotated about the x-axis, then the **volume** of the generated solid is equal to

Using cylindrical shells



(a) $\frac{\pi}{2}$

(b) $\frac{3\pi}{4}$

(c) π

(d) $\frac{5\pi}{4}$

(e) $\frac{3\pi}{2}$

shell radius $= r = y$

shell height $= h = y - y^2 - (y-1)$
$= 1 - y^2$

$$V = \int_0^1 2\pi r h\, dy$$

$$= 2\pi \int_0^1 y(1-y^2)\,dy = 2\pi \int_0^1 (y - y^3)\,dy$$

$$= 2\pi \left[\frac{1}{2}y^2 - \frac{1}{4}y^4\right]_0^1 = 2\pi\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{\pi}{2}.$$

9. $\int x^2 \sin x\, dx =$ I . By Parts,

$$\begin{array}{lcl} & & \sin x \\ x^2 & \xrightarrow{(+)} & -\cos x \\ 2x & \xrightarrow{(-)} & -\sin x \\ 2 & \xrightarrow{(+)} & \cos x \\ 0 & & \sin x \end{array}$$

(a) $-x^2\cos x + 2x\sin x + 2\cos x + C$

(b) $x^2\cos x + x\cos x + \cos x + C$

(c) $x^2\cos x - 2x\sin x - 2\cos x + C$

(d) $-x^2\sin x - 2x\sin x - \cos x + C$

(e) $x^2\sin x + 2x\cos x - 2\sin x + C$

$$I = x^2(-\cos x) - 2x(-\sin x) + 2\cos x + C$$
$$= -x^2\cos x + 2x\sin x + 2\cos x + C$$

10. $\int \frac{\sin\sqrt{\theta}}{\sqrt{\theta\cos^3\sqrt{\theta}}}\, d\theta =$ I

Let $u = \cos\sqrt{\theta}$

$$du = -\sin\sqrt{\theta}\left(\frac{1}{2\sqrt{\theta}}\right)d\theta = -\frac{1}{2}\,\frac{\sin\sqrt{\theta}}{\sqrt{\theta}}\,d\theta$$

(a) $\frac{4}{\sqrt{\cos\sqrt{\theta}}} + C$

(b) $\frac{2}{\sqrt{\cos\sqrt{\theta}}} + C$

(c) $\frac{2\sqrt{\theta}}{\sqrt{\cos\sqrt{\theta}}} + C$

(d) $\frac{\sqrt{\theta}}{\sqrt{\sin^3\sqrt{\theta}}} + C$

(e) $\frac{-4}{\sqrt{\cos\sqrt{\theta}}} + C$

$$I = \int \frac{1}{\sqrt{\cos^3\sqrt{\theta}}}\;\frac{\sin\sqrt{\theta}}{\sqrt{\theta}}\,d\theta$$
$$= \int \frac{1}{\sqrt{u^3}}\,(-2\,du) = -2\int u^{-3/2}\,du$$
$$= -2\cdot(-2)u^{-1/2} + C$$
$$= \frac{4}{\sqrt{u}} + C = \frac{4}{\sqrt{\cos\sqrt{\theta}}} + C.$$

11. Let $f(x) = \int_{-1}^{x} t^{10} \cdot \tan\left(\frac{\pi t}{4}\right) dt$. An equation for the **tangent line** to the graph of f at $x = 1$ is

(a) $y = x - 1$

(b) $y = x + 2$

(c) $y = 3x + 1$

(d) $y = x - 3$

(e) $y = -x + 1$

$$f'(x) = \frac{d}{dx} \int_{-1}^{x} t^{10} \cdot \tan\left(\frac{\pi t}{4}\right) dt$$

$$= x^{10} \tan\left(\frac{\pi x}{4}\right)$$

$$f'(1) = \tan\left(\frac{\pi}{4}\right) = 1.$$

$$f(1) = \int_{-1}^{1} t^{10} \cdot \tan\left(\frac{\pi t}{4}\right) dt$$

$= 0.$ ($t^{10} \tan\left(\frac{\pi t}{4}\right)$ is an odd function)

Thus, an equation of the tangent line is

$$y - f(1) = f'(1)(x-1)$$

$$\Rightarrow y - 0 = 1(x-1)$$

$$\Rightarrow y = x - 1.$$

12. $\int_0^1 [\ln(\cosh x + \sinh x)^3 + \ln(\cosh x - \sinh x)^2]\, dx =$

(a) $\frac{1}{2}$

(b) 1

(c) 0

(d) $\frac{3}{2}$

(e) 5

$$\int_0^1 \left[\ln\left(\frac{e^x + e^{-x} + e^x - e^{-x}}{2}\right)^3 + \ln\left(\frac{e^x + e^{-x} - e^x + e^{-x}}{2}\right)^2\right] dx$$

$$= \int_0^1 \left[\ln(e^x)^3 + \ln(e^{-x})^2\right] dx$$

$$= \int_0^1 \left(\ln e^{3x} + \ln e^{-2x}\right) dx$$

$$= \int_0^1 (3x - 2x)\, dx$$

$$= \int_0^1 x\, dx = \frac{1}{2} x^2 \Big]_0^1 = \frac{1}{2}.$$

13. $\int_{1/2}^{1} \frac{(1-x^2)^{3/2}}{x^6}\,dx = I.$

(a) $\frac{9\sqrt{3}}{5}$

(b) $\frac{7\sqrt{3}}{5}$

(c) $\sqrt{3}$

(d) $\frac{3\sqrt{3}}{5}$

(e) $\frac{2\sqrt{3}}{7}$

Let $x = \sin\theta \Rightarrow 1-x^2 = \cos^2\theta$

$dx = \cos\theta\, d\theta$

$x = \frac{1}{2} \Rightarrow \theta = \sin^{-1}\frac{1}{2} = \pi/6$

$x = 1 \Rightarrow \theta = \sin^{-1} 1 = \pi/2$

$$I = \int_{\pi/6}^{\pi/2} \frac{(\cos^2\theta)^{3/2}}{\sin^6\theta}\cos\theta\, d\theta$$

$$= \int_{\pi/6}^{\pi/2} \frac{\cos^3\theta\cos\theta}{\sin^6\theta}\,d\theta = \int_{\pi/6}^{\pi/2} \frac{\cos^4\theta}{\sin^4\theta}\frac{1}{\sin^2\theta}\,d\theta$$

$$= \int_{\pi/6}^{\pi/2} \cot^4\theta\,\csc^2\theta\,d\theta = -\int_{\pi/6}^{\pi/2} \cot^4\theta\, d(\cot\theta)$$

$$= -\frac{1}{5}\cot^5\theta\Big]_{\pi/6}^{\pi/2}$$

$$= -\frac{1}{5}\cot^5\left(\frac{\pi}{2}\right) + \frac{1}{5}\cot^5(\pi/6)$$

$$= 0 + \frac{1}{5}(\sqrt{3})^5 = \frac{9\sqrt{3}}{5}.$$

14. The improper integral $\int_0^1 \frac{1}{\sqrt{|2x-1|}}\,dx$ is $= I$

(a) convergent and its value is 2

(b) convergent and its value is 1

(c) convergent and its value is 3

(d) convergent and its value is 4

(e) divergent

$$|2x-1| = \begin{cases} 2x-1, & x \geq \frac{1}{2} \\ 1-2x, & x < \frac{1}{2} \end{cases}$$

$$I = \int_0^{1/2} \frac{1}{\sqrt{1-2x}}\,dx + \int_{1/2}^{1} \frac{1}{\sqrt{2x-1}}\,dx$$

$$= \lim_{t\to\frac{1}{2}^-} \int_0^t (1-2x)^{-1/2}\,dx + \lim_{t\to\frac{1}{2}^+} \int_t^1 (2x-1)^{-1/2}\,dx$$

$$= \lim_{t\to\frac{1}{2}^-} \left[-\frac{1}{2}2\sqrt{1-2x}\right]_0^t + \lim_{t\to\frac{1}{2}^+} \left[\frac{1}{2}2\sqrt{2x-1}\right]_t^1$$

$$= \lim_{t\to\frac{1}{2}^-} \left(-\sqrt{1-2t}+1\right) + \lim_{t\to\frac{1}{2}^+} \left(1-\sqrt{2t-1}\right)$$

$$= (-0+1) + (1-0) = 2.$$

15. The series $\sum_{n=1}^{\infty}\left[\sin^{-1}\left(\frac{n}{n+1}\right)-\sin^{-1}\left(\frac{n+1}{n+2}\right)\right]$ is

(a) convergent and its sum is $\frac{-\pi}{3}$

(b) convergent and its sum is $\frac{\pi}{6}$

(c) convergent and its sum is π

(d) convergent and its sum is $\frac{-\pi}{4}$

(e) divergent

$$S_n = \left(\sin^{-1}\tfrac{1}{2} - \cancel{\sin^{-1}\tfrac{2}{3}}\right) + \left(\cancel{\sin\tfrac{2}{3}} - \cancel{\sin\tfrac{3}{4}}\right) + \cdots + \left(\cancel{\sin\left(\tfrac{n-1}{n}\right)} - \cancel{\sin^{-1}\left(\tfrac{n}{n+1}\right)}\right) + \left(\cancel{\sin^{-1}\left(\tfrac{n}{n+1}\right)} - \sin^{-1}\left(\tfrac{n+1}{n+2}\right)\right)$$

$$= \sin^{-1}\tfrac{1}{2} - \sin^{-1}\left(\tfrac{n+1}{n+2}\right)$$

$$\lim_{n\to\infty} S_n = \sin^{-1}\left(\tfrac{1}{2}\right) - \sin^{-1}(1) = \frac{\pi}{6} - \frac{\pi}{2} = \frac{-2\pi}{6} = -\frac{\pi}{3}.$$

16. The series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{4^{n-1}}{3^{n+1}}$ is

$$= \sum_{n=1}^{\infty}(-1)^{n-1}\frac{4^{n-1}}{3^2\,3^{n-1}} = \sum_{n=1}^{\infty}\frac{1}{9}\left(\frac{-4}{3}\right)^{n-1}$$

which is a divergent geometric series with $|r| = \frac{4}{3} > 1$.

(a) divergent

(b) convergent and its sum is $-\frac{1}{4}$

(c) convergent and its sum is $-\frac{3}{7}$

(d) convergent and its sum is $\frac{3}{5}$

(e) convergent and its sum is -3

OR

$$u_n = \frac{4^{n-1}}{3^{n+1}} = \frac{4^{-1}}{3}\,\frac{4^n}{3^n} = \frac{1}{12}\left(\frac{4}{3}\right)^n \longrightarrow \infty$$

as $n \longrightarrow \infty$

so the series diverges by the nth-term test for divergence.

17. Let $a_n = \int_n^{n+1} \frac{1}{t}\, dt$. Then the sequence $\{a_n\}_{n=1}^{\infty}$ is

(a) convergent to 0

(b) convergent to 1

(c) convergent to $\ln n$

(d) convergent to $\ln 2$

(e) divergent

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \int_n^{n+1} \frac{1}{t}\, dt$$
$$= \lim_{n\to\infty} \left[\ln|t|\right]_n^{n+1}$$
$$= \lim_{n\to\infty} (\ln(n+1) - \ln n)$$
$$= \lim_{n\to\infty} \ln\left(\frac{n+1}{n}\right)$$
$$= \ln(1) = 0.$$

18. The Taylor polynomial of order 3 generated by $f(x) = \ln(x-1)$ at $x = 2$ is the polynomial $P_3(x) = f(2) + f'(2)(x-2) + \frac{f''(2)}{2!}(x-2)^2 + \frac{f^{(3)}(2)}{3!}(x-2)^3$

(a) $(x-2) - \frac{1}{2}(x-2)^2 + \frac{1}{3}(x-2)^3$

(b) $1 + (x-2) + \frac{1}{2}(x-2)^2 + \frac{1}{6}(x-2)^3$

(c) $-2(x-2) + \frac{1}{3}(x-2)^2 - \frac{1}{2}(x-2)^3$

(d) $\frac{1}{2} - (x-2) - \frac{1}{2}(x-2)^2 - \frac{1}{3}(x-2)^3$

(e) $3(x-2) - \frac{1}{4}(x-2)^2 + \frac{1}{3}(x-2)^3$

$f(2) = \ln(1) = 0$

$f'(x) = \frac{1}{x-1}$, $f'(2) = 1$

$f''(x) = \frac{-1}{(x-1)^2}$, $f''(2) = -1$

$f^{(3)}(x) = \frac{2}{(x-1)^3}$, $f^{(3)}(2) = 2$.

Thus,

$$P_3(x) = 0 + 1(x-2) + \frac{-1}{2}(x-2)^2 + \frac{2}{6}(x-2)^3$$
$$= (x-2) - \frac{1}{2}(x-2)^2 + \frac{1}{3}(x-2)^3.$$

19. The series $\sum_{n=1}^{\infty} \frac{e^{n^2}}{n^n}$ is

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \left(\frac{e^{n^2}}{n^n}\right)^{\frac{1}{n}}$$

$$= \lim_{n\to\infty} \frac{e^n}{n}.$$

(a) divergent by the root test

(b) convergent by the root test

Since $\lim_{x\to\infty} \frac{e^x}{x} \overset{H}{=} \lim_{x\to\infty} \frac{e^x}{1}$

(c) a series for which the root test is inconclusive

$= \infty > 1.$

(d) convergent by the ratio test

The series diverges by the root test.

(e) divergent by the alternating series test.

20. Using the **binomial series,** we have for $|x| < 1$, $\sqrt{1-x} =$

$$(1-x)^{\frac{1}{2}} = 1 + \tfrac{1}{2}(-x) + \frac{(\frac{1}{2})(\frac{1}{2}-1)}{2!}(-x)^2$$

$$+ \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(-x)^3$$

$$+ \cdots, \quad |-x| < 1.$$

(a) $1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 + \ldots$

(b) $1 + \frac{1}{2}x + \frac{1}{3}x^2 + \frac{1}{4}x^3 + \ldots$

(c) $1 - \frac{1}{3}x + \frac{1}{4}x^2 - \frac{1}{5}x^3 + \ldots$

(d) $1 - x + x^2 - x^3 + \ldots$

(e) $1 - \frac{1}{2}x + \frac{1}{8}x^2 - \frac{1}{16}x^3 + \ldots$

$$= 1 - \frac{1}{2}x + \frac{\frac{1}{2}(-\frac{1}{2})}{2}x^2$$

$$+ \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{6}(-x^3)$$

$$+ \cdots$$

$$= 1 - \tfrac{1}{2}x - \tfrac{1}{8}x^2 - \tfrac{1}{16}x^3 + \cdots.$$

21. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+\sqrt{n}}$ is

(a) absolutely convergent

(b) conditionally convergent

(c) divergent

(d) neither convergent nor divergent

(e) absolutely and conditionally convergent

$\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{n^2+\sqrt{n}}$ is convergent by the Comparison test with the convergent p-series $\sum_{n=1}^{\infty} \frac{1}{n^2}$, because

$|a_n| = \frac{1}{n^2+\sqrt{n}} < \frac{1}{n^2}$.

So, $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+\sqrt{n}}$ is absolutely Convergent.

22. Let $a_n > 0$ for $n \geq 1$. Which one of the following statements is **TRUE:**

(a) If $\lim_{n\to\infty} \frac{a_n}{a_{n+1}} = 2$, then $\sum_{n=1}^{\infty} a_n$ converges.

$\lim_{n\to 1} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{1}{a_n/a_{n+1}} = \frac{1}{2}$

$\sum_{n=1}^{\infty} a_n$ converges by the ratio test.

(b) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} \sqrt{a_n}$ converges. Take, $\sum_{n=1}^{\infty} \frac{1}{n^2}$, $\sum_{n=1} \frac{1}{n}$.

(c) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} \sqrt{a_n}$ converges. Take, $\sum_{n=1}^{\infty} \frac{1}{n}$, $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.

(d) If $\lim_{n\to\infty} \sqrt[n]{a_n} = 1$, then $\sum a_n$ diverges Take, $\sum_{n=1}^{\infty} e^{-n^2}$.

(e) If $a_n \leq \frac{1}{n}$ for $n \geq 1$, then $\sum_{n=1}^{\infty} a_n$ diverges Take, $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

23. The **interval of convergence** of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{3n}(x-1)^n$ is

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(-1)^{n+1}2^{n+1}(x-1)^{n+1}\,3n}{3(n+1)(-1)^n 2^n (x-1)^n}\right|$$

(a) $\left(\frac{1}{2}, \frac{3}{2}\right]$

$$= \lim_{n\to\infty} \frac{2n}{n+1}|x-1| = 2|x-1|$$

(b) $\left[\frac{1}{2}, \frac{3}{2}\right)$

By the Ratio Test, the series converges for $2|x-1|<1 \Rightarrow |x-1|<\frac{1}{2}$

(c) $\left(\frac{1}{2}, \frac{3}{2}\right)$

$\Rightarrow \frac{1}{2}<x<\frac{3}{2}$.

(d) $\left[\frac{-1}{2}, \frac{1}{2}\right]$

$\underline{x=\frac{1}{2}}$: $\sum_{n=1}^{\infty}\frac{1}{3n} = \frac{1}{3}\sum_{n=1}^{\infty}\frac{1}{n}$ divergent series

(e) $\left(\frac{-1}{2}, \frac{1}{2}\right)$

$\underline{x=\frac{3}{2}}$: $\sum_{n=1}^{\infty}\frac{(-1)^n}{3n} = \frac{1}{3}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ convergent alternating series.

So, the interval of convergent is $\left(\frac{1}{2}, \frac{3}{2}\right]$.

24. The series $\sum_{n=1}^{\infty}\left(\frac{\cos n}{n+3}\right)^2$ is

$0 \le \cos^2 n \le 1$

(a) convergent by the comparison test $\Rightarrow \frac{\cos^2 n}{(n+3)^2} \le \frac{1}{(n+3)^2}$

(b) a convergent geometric series $\le \frac{1}{n^2}$

(c) divergent by the integral test , $\sum_{n=1}^{\infty}\frac{1}{n^2}$ is convergent p-series $(p=2)$

(d) divergent by the limit comparison test

(e) divergent by the n^{th} – term test for divergence

Thus, $\sum_{n=0}^{\infty}\frac{\cos^2 n}{(n+3)^2}$ is convergent by the Comparison test.

25. The first three nonzero terms of the Maclaurin series of the function $(x-1)\left(\frac{e^x-1-x}{\cancel{x^2}}\right)$ are:

(a) $-\frac{x^2}{2}+\frac{1}{3}x^3+\frac{1}{8}x^4$

(b) $-\frac{1}{2}-\frac{1}{3}x-\frac{1}{8}x^2$

(c) $\frac{1}{2}-\frac{1}{6}x+\frac{1}{24}x^2$

(d) $\frac{1}{2}x+\frac{1}{6}x^2+\frac{1}{24}x^2$

(e) $-\frac{1}{2}-\frac{1}{6}x+\frac{1}{3}x^2$

$$(x-1)(e^x-1-x)$$
$$=(x-1)\left(\cancel{1}+\cancel{x}+\frac{1}{2!}x^2+\frac{1}{3!}x^3+\frac{1}{4!}x^4+\cdots\cancel{-1}\cancel{-x}\right)$$
$$=(x-1)\left(\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{24}x^4+\cdots\right)$$
$$=-\frac{1}{2}x^2-\frac{1}{6}x^3-\frac{1}{24}x^4-\cdots+\frac{1}{2}x^3+\frac{1}{6}x^4+\cdots$$
$$=-\frac{1}{2}x^2+\left(\frac{1}{2}-\frac{1}{6}\right)x^3+\left(\frac{1}{6}-\frac{1}{24}\right)x^4+\cdots$$

So, the first three nonzero terms are $-\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{1}{8}x^4$.

26. For some suitable values of x, the Maclaurin series for $f(x)=\frac{1}{(3-x)^2}$ is given by

(a) $\sum_{n=1}^{\infty}\frac{n}{3^{n+1}}x^{n-1}$

(b) $\sum_{n=1}^{\infty} n\,x^{n-1}$

(c) $\sum_{n=1}^{\infty} n\cdot 3^n\cdot x^n$

(d) $\sum_{n=1}^{\infty} 3^n\,x^{n-1}$

(e) $\sum_{n=1}^{\infty}\frac{n}{3^n}x^{n-1}$

$$\frac{1}{(3-x)^2}=\frac{d}{dx}\left(\frac{1}{3-x}\right)=\frac{d}{dx}\left(\frac{1}{3}\,\frac{1}{1-\frac{x}{3}}\right)$$
$$=\frac{1}{3}\,\frac{d}{dx}\left(\frac{1}{1-\frac{x}{3}}\right)$$
$$=\frac{1}{3}\,\frac{d}{dx}\left(\sum_{n=0}^{\infty}\left(\frac{x}{3}\right)^n\right),\ \left|\frac{x}{3}\right|<1.$$
$$=\frac{1}{3}\sum_{n=1}^{\infty}n\left(\frac{x}{3}\right)^{n-1}\cdot\left(\frac{1}{3}\right),\ |x|<\frac{1}{3}$$
$$=\sum_{n=1}^{\infty}\frac{n}{3^{n+1}}x^{n-1}.$$

27. The sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{36^{n-1}} \cdot \frac{\pi^{2n}}{(2n)!}$ is equal to

(a) $18\sqrt{3}$

(b) $9\sqrt{3}$

(c) $6\sqrt{3}$

(d) 12

(e) 15

Since $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$, $|x| < \infty$,

then $\sum_{n=0}^{\infty} \frac{(-1)^n}{36^{n-1}} \frac{\pi^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n}{6^{2n-2}} \frac{\pi^{2n}}{(2n)!}$

$= 6^2 \sum_{n=0}^{\infty} (-1)^n \frac{\left(\frac{\pi}{6}\right)^{2n}}{(2n)!}$

$= 36 \cos\left(\frac{\pi}{6}\right)$

$= 36\left(\frac{\sqrt{3}}{2}\right)$

$= 18\sqrt{3}$.

28. $\int_0^1 \sin(x^5)\, dx =$ $\int_0^1 \sum_{n=0}^{\infty} (-1)^n \frac{(x^5)^{2n+1}}{(2n+1)!}\, dx$

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)! \cdot (10n+6)}$

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)! \cdot (2n+2)}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)! \cdot (5n+4)}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!(10n+3)}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!(6n+1)}$

$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \int_0^1 x^{10n+5}\, dx$

$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \left[\frac{x^{10n+6}}{10n+6} \right]_0^1$

$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!\,(10n+6)}$.