KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
MATH 102 - QUIZ 2

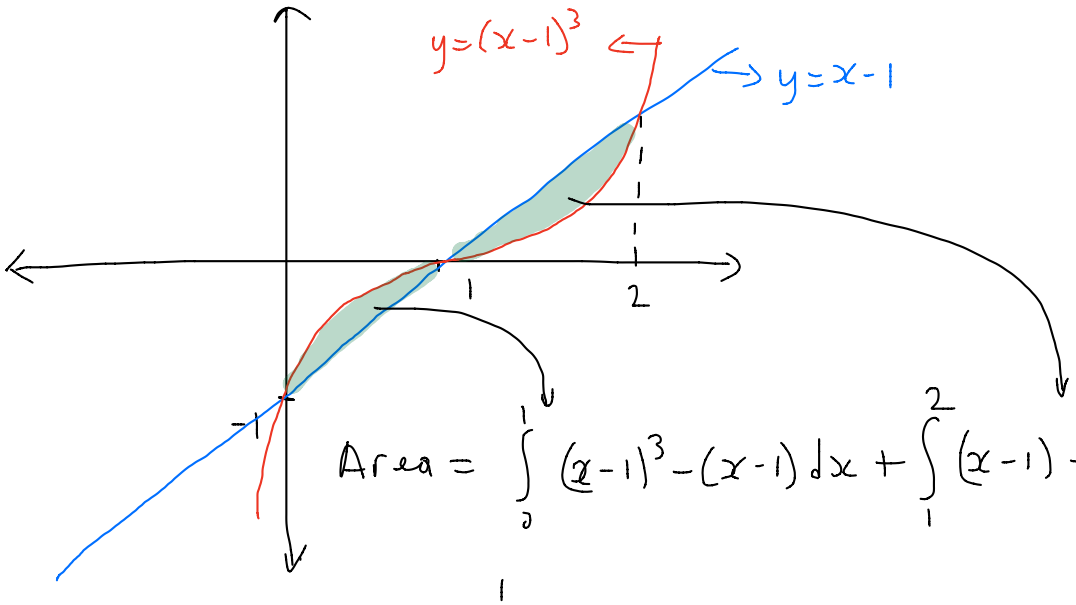
Name:
Student ID #:

Question 1. Find the area enclosed by $y = (x-1)^3$ and $y = x - 1$.

Question 2. Evaluate the indefinite integral $\int \frac{x^3}{\sqrt{1-x^2}} dx$.

Your Solution.

Question 1



$$\text{Area} = \int_0^1 (x-1)^3 - (x-1)\,dx + \int_1^2 (x-1) - (x-1)^3\,dx$$

$$= \int_0^1 x^3 - 3x^2 + 3x - 1 - x + 1\,dx + \int_1^2 x - 1 - (x^3 - 3x^2 + 3x - 1)\,dx$$

$$= \int_0^1 x^3 - 3x^2 + 2x\,dx + \int_1^2 -x^3 + 3x^2 - 2x\,dx$$

$$= \frac{x^4}{4} - x^3 + x^2 \Big|_0^1 + \left(-\frac{x^4}{4} + x^3 - x^2\right)\Big|_1^2$$

$$= \left(\frac{1}{4} - 1 + 1\right) + \left(-4 + 8 - 4\right) - \left(-\frac{1}{4} + 1 - 1\right) = \frac{1}{2} \;//$$

Question 2

$$\int \frac{x^3}{\sqrt{1-x^2}}\,dx = \int \frac{x\,x^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\int \frac{1-u}{\sqrt{u}}\,du = -\frac{1}{2}\int \frac{1-u}{u^{1/2}}\,du$$

Do the substitution:

$u = 1-x^2 \Rightarrow du = -2x\,dx$

$\Rightarrow \frac{du}{-2} = x\,dx$

$x^2 = 1-u$

$$= -\frac{1}{2}\int u^{-1/2} - u^{1/2}\,du = -\frac{1}{2}\left[2u^{1/2} - \frac{2}{3}u^{3/2}\right] + C$$

$$= \frac{1}{3}u^{3/2} - u^{1/2} + C = \frac{1}{3}(1-x^2)^{3/2} - (1-x^2)^{1/2} + C$$