1. If $F(x) = \int_1^{e^{x^2}} \cos(\pi \ln t)\, dt$ then $F'(1) =$

(a) $-2e$

(b) 0

(c) -1

(d) 1

(e) e^2

$$F'(x) = \frac{d}{dx}\left(\int_1^{e^{x^2}} \cos(\pi \ln t)\, dt\right)$$
$$= \cos(\pi \ln e^{x^2}) \cdot 2x e^{x^2}$$
$$= 2x e^{x^2} \cos(\pi x^2)$$
$$F'(1) = 2e^1 \cos(\pi) = -2e.$$

2. $\int \sqrt{\sin x} \cos^5 x\, dx = \int \sin^{1/2} x \cos^4 x \cos x\, dx = \int \sin^{1/2} x (1 - \sin^2 x)^2 \cos x\, dx$

$= \int \sin^{1/2} x (1 - 2\sin^2 x + \sin^4 x) \cos x\, dx$

$= \int (u^{1/2} - 2u^{5/2} + u^{9/2})\, du$ (where $u = \sin x$, $du = \cos x\, dx$)

$= \frac{2}{3} u^{3/2} - \frac{4}{7} u^{7/2} + \frac{2}{11} u^{11/2} + C$, $u = \sin x$.

(a) $\frac{2}{3} \sin^{3/2} x - \frac{4}{7} \sin^{7/2} x + \frac{2}{11} \sin^{11/2} x + c$

(b) $\sin^{3/2} x - \frac{2}{3} \sin^{7/2} x + \frac{2}{11} \sin^{11/2} x + c$

(c) $2\sin^{1/2} x - 3\sin^{7/2} x + 11 \sin^{11/2} x + c$

(d) $\cos^{5/2} x - \frac{3}{2} \sin x \cos^6 x + \frac{5}{6} \cos^5 x \cos x + c$

(e) $\sin^{1/2} x \cos^4 x + \sin x \cos^{5/2} x + \frac{3}{4} \cos^4 x \sin^{3/2} x + c$

3. The area of the region between the parabola $y = x^2$ and the line $y = x$ from $x = 0$ to $x = 2$ is equal to

(a) 1

(b) $\frac{3}{2}$

(c) $\frac{1}{6}$

(d) 2

(e) $\frac{5}{6}$

$x = x^2 \Rightarrow x(x-1) = 0$

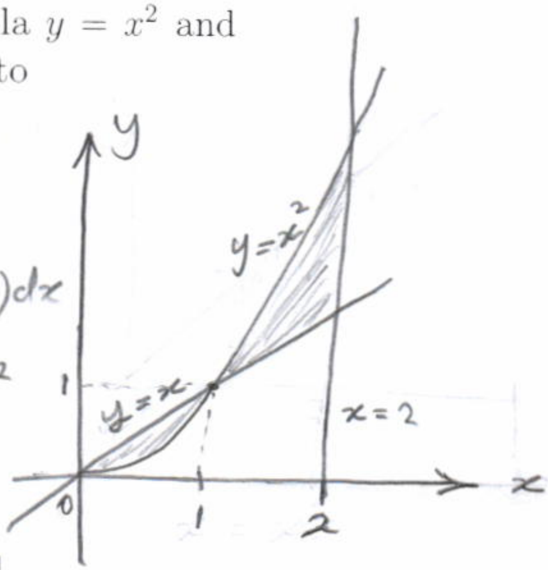
$\Rightarrow x = 0, 1$

$A = \int_0^1 (x - x^2)\,dx + \int_1^2 (x^2 - x)\,dx$

$= \left[\frac{1}{2}x^2 - \frac{1}{3}x^3\right]_0^1 + \left[\frac{1}{3}x^3 - \frac{1}{2}x^2\right]_1^2$

$= \frac{1}{2} - \frac{1}{3} + \frac{8}{3} - 2 - \frac{1}{3} + \frac{1}{2}$

$= -1 + \frac{6}{3} = 1.$



4. The sequence $a_n = \ln n - \ln(n-1)$

(a) converges to 0

(b) diverges

(c) converges to 1

(d) converges to -1

(e) converges to 5

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \ln n - \ln(n-1)$

$= \lim_{n\to\infty} \ln \frac{n}{n-1}$

$= \ln\left(\lim_{n\to\infty} \frac{n}{n-1}\right)$

$= \ln(1)$

$= 0.$

5. The length of the curve of $y = \frac{1}{3}(x^2+2)^{3/2}$ from $x=0$ to $x=3$ is equal to

(a) 12

(b) 10

(c) 8

(d) 6

(e) 4

$$\frac{dy}{dx} = \frac{1}{2}(x^2+2)^{1/2} \cdot (2x) = x\sqrt{x^2+2}$$

$$1+\left(\frac{dy}{dx}\right)^2 = 1 + x^2(x^2+2)$$

$$L = \int_0^3 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_0^3 \sqrt{x^4+2x^2+1}\,dx$$

$$= \int_0^3 \sqrt{(x^2+1)^2}\,dx = \int_0^3 (x^2+1)\,dx$$

$$= \left[\frac{1}{3}x^3+x\right]_0^3 = 9+3 = 12.$$

6. The improper integral $\int_1^2 \frac{dx}{(x-1)^{4/3}}$

(a) diverges

(b) converges to 1

(c) converges to 2

(d) converges to $\sqrt[3]{2}$

(e) converges to -3

$$= \lim_{t\to 1^+} \int_t^2 (x-1)^{-4/3}\,dx$$

$$= \lim_{t\to 1^+} -3(x-1)^{-1/3}\Big]_t^2$$

$$= \lim_{t\to 1^+} -3(1)^{-1/3} + 3(t-1)^{-1/3}$$

$$= -3 + 3\lim_{t\to 1^+} \frac{1}{(t-1)^{1/3}}$$

$$= -3 + \infty = \infty.$$

The integral diverges.

7. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$ is

$b_n = \frac{1}{n^2} > 0,$

$\frac{1}{(n+1)^2} < \frac{1}{n^2}$ for all $n \geq 1$

(a) Convergent by the alternating series test

and

(b) Divergent by the alternating series test

$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{n^2} = 0.$

(c) Divergent by the root test

So the series converges by the alternating series test.

(d) Convergent by the integral test

(e) A series where the alternating series test is inconclusive.

8. $\sum_{n=0}^{\infty} \left(\frac{3}{4^n} + \frac{1}{3^n}\right) =$

(a) $\frac{11}{2}$

(b) $\frac{3}{4}$

(c) $\frac{5}{2}$

(d) $\frac{7}{2}$

(e) 4

- $\sum_{n=0}^{\infty} \frac{3}{4^n} = 3 \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$ convergent geometric series with $a = 1$, $r = \frac{1}{4}$.
  Its sum is $\frac{1}{1-1/4} = 4/3$
- Also, $\sum_{n=0}^{\infty} \frac{1}{3^n}$ is a convergent geometric series with $a = 1$, $r = \frac{1}{3}$.
  Its sum is $\frac{1}{1-1/3} = 3/2$

So $\sum_{n=0}^{\infty} \left(\frac{3}{4^n} + \frac{1}{3^n}\right) = 3\sum_{n=0}^{\infty} \frac{1}{4^n} + \sum_{n=0}^{\infty} \frac{1}{3^n}$

$= 3\left(\frac{4}{3}\right) + \frac{3}{2} = \frac{11}{2}.$

9. The series $\sum_{n=1}^{\infty} (\tan^{-1} n)^n$ is

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \left|(\tan^{-1} n)^n\right|^{1/n}$$

$$= \lim_{n\to\infty} \tan^{-1} n$$

$$= \frac{\pi}{2} > 1$$

The series diverges by the root test.

(a) Divergent by the root test

(b) Convergent by the root test

(c) Convergent by the ratio test

(d) A series where the root test is inconclusive

(e) Convergent by the comparison test

10. $\displaystyle\int \frac{x\cos x^2 + 1 + \sin^2 x^2}{1+\sin^2 x^2}\,dx =$ $\displaystyle\int \left(\frac{x\cos x^2}{1+\sin^2 x^2} + 1\right) dx$

$$= \int \frac{x\cos x^2}{1+(\sin x^2)^2}\,dx + x + c$$

$$\begin{cases} u = \sin(x^2) \\ du = 2x\cos(x^2)\,dx \end{cases}$$

$$= \frac{1}{2}\int \frac{du}{1+u^2} + x + c$$

$$= \frac{1}{2}\tan^{-1} u + x + c$$

$$= \frac{1}{2}\tan^{-1}(\sin(x^2)) + x + c.$$

(a) $\frac{1}{2}\tan^{-1}(\sin x^2) + x + c$

(b) $\tan^{-1}(\sin x^2) + x + c$

(c) $2\tan^{-1}(\sin x^2) + x + c$

(d) $-\frac{1}{2}\tan^{-1}(\sin x^2) + c$

(e) $-2\tan^{-1}(\sin x) + 3x + c$

11. $\int e^{2x} \sin x \, dx = I$.

By Parts

$u = e^{2x}$ $dv = \sin x \, dx$

$du = 2e^{2x} dx$ $v = -\cos x$

$I = -e^{2x}\cos x + 2\int e^{2x}\cos x \, dx$

$\begin{cases} u = e^{2x} & dv = \cos x \, dx \\ du = 2e^{2x} dx & v = \sin x \end{cases}$

$= -e^{2x}\cos x + 2\left(e^{2x}\sin x - 2\int e^{2x}\sin x \, dx\right)$

$= -e^{2x}\cos x + 2e^{2x}\sin x - 4I$

Thus,

$5I = -e^{2x}\cos x + 2e^{2x}\sin x$

and $I = -\frac{1}{5}e^{2x}\cos x + \frac{2}{5}e^{2x}\sin x$.

(a) $-\dfrac{1}{5}e^{2x}\cos x + \dfrac{2}{5}e^{2x}\sin x + c$

(b) $e^{2x}\cos x + \dfrac{2}{5}e^{2x}\sin x + c$

(c) $\dfrac{1}{5}e^{2x}\sin x + \dfrac{2}{5}e^{2x}\cos x + c$

(d) $\dfrac{3}{5}e^{2x}\cos x - \dfrac{2}{5}e^{2x}\sin x + c$

(e) $\dfrac{4}{5}e^{2x}\sin x - \dfrac{3}{5}e^{2x}\cos x + c$

12. $\int_0^1 \frac{x\sqrt{x}}{(1+x^2\sqrt{x})^2} \, dx = \int_0^1 \frac{x^{3/2}}{(1+x^{5/2})^2} dx$

Let $u = 1 + x^{5/2}$

$du = \frac{5}{2}x^{3/2} dx$

x	0	1
u	1	2

$= \int_1^2 \frac{2/5 \, du}{u^2} = \frac{2}{5}\int_1^2 u^{-2} du$

$= \frac{2}{5}\left[-u^{-1}\right]_1^2 = \frac{2}{5}\left(-\frac{1}{2}+1\right) = \frac{1}{5}$.

(a) $\dfrac{1}{5}$

(b) $\dfrac{1}{2}$

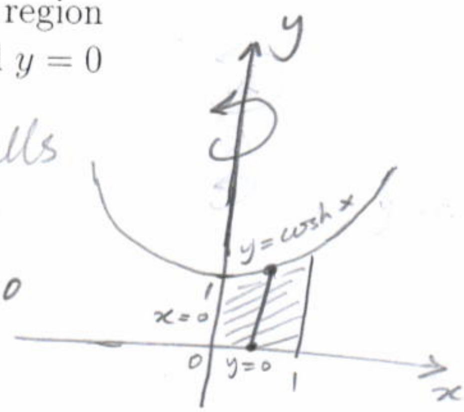
(c) $\dfrac{4}{5}$

(d) $\dfrac{3}{2}$

(e) 1

13. The volume of the solid generated by rotating the region bounded by the curves $y = \cosh x$, $x = 0$, $x = 1$ and $y = 0$ about the y-axis is given by

By cylindrical shells



(a) $2\pi \int_0^1 x \cosh x \, dx$

(b) $\pi \int_0^1 x \cosh^2 x - x^2 \, dx$

(c) $2\pi \int_0^1 x^2 \cosh x \, dx$

(d) $2\pi \int_0^1 x \cosh(2x) \, dx$

(e) $\pi \int_0^1 x (\cosh^2 x - 4x^2) \, dx$

radius $= r = x$
height $= h = \cosh x - 0$
$a = 0, b = 1$

$V = \int_a^b 2\pi r h \, dx$

$= 2\pi \int_0^1 x \cosh x \, dx.$

14. $\int \frac{3x^2 - 5x + 13}{(x-2)(x^2+1)} \, dx = I.$ By partial fractions.

$$\frac{3x^2 - 5x + 13}{(x-2)(x^2+1)} = \frac{A}{x-2} + \frac{Bx + C}{x^2+1}$$

$\Rightarrow 3x^2 - 5x + 13 = A(x^2+1) + (Bx+C)(x-2)$

(a) $3 \ln|x-2| - 5\tan^{-1} x + c$

(b) $3 \ln|x-2| + \ln(x^2+1) - 5\tan^{-1} x + c$

(c) $2 \ln|x-2| + 4 \ln(x^2+1) + c$

(d) $3 \ln|x-2| + 10\tan^{-1} x + c$

(e) $3 \ln|x-2| + 4\ln(x^2+1) - 10\tan^{-1} x + c$

$\underline{x = 2}$: $12 - 10 + 13 = 5A$
$\Rightarrow \boxed{A = 3}$

$\underline{\text{coef. } x^2}$: $3 = A + B \Rightarrow \boxed{B = 0}$

$\underline{\text{Constant term}}$: $13 = A - 2C \Rightarrow \boxed{C = -5}$

$I = \int \frac{3}{x-2} dx + \int \frac{-5}{x^2+1} dx$

$= 3 \ln|x-2| - 5\tan^{-1} x + C$

15. The series $\sum_{n=1}^{\infty}(-1)^n \frac{10^n}{(n+1)!}$ is

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{10^{n+1}}{(n+2)!}\frac{(n+1)!}{10^n}\right|$

$= \lim_{n\to\infty}\frac{10}{n+2} = 0 < 1$

The series is absolutely convergent by the ratio test.

(a) Absolutely convergent

(b) Divergent

(c) Conditionally covergent

(d) Divergent by the nth term test of divergence

(e) Convergent by the comparison test

16. $\int_0^{\sqrt{3}/2} \frac{1}{(1-x^2)^{5/2}}\, dx =$

let $x = \sin\theta$

$dx = \cos\theta \, d\theta$

$(1-x^2)^{5/2} = (1-\sin^2 x)^{5/2} = \cos^5 x$.

x	0	$\sqrt{3}/2$
θ	0	$\pi/3$

(a) $2\sqrt{3}$

(b) $\frac{3\sqrt{3}}{2}$

(c) $3\sqrt{3}$

(d) $\frac{2\sqrt{3}}{3}$

(e) $\frac{\sqrt{3}}{6}$

$= \int_0^{\pi/3} \frac{\cos\theta\, d\theta}{\cos^5\theta} = \int_0^{\frac{\pi}{3}} \frac{1}{\cos^4\theta}\, d\theta$

$= \int_0^{\pi/3} \sec^4\theta\, d\theta = \int_0^{\pi/3} \sec^2\theta \sec^2\theta\, d\theta$

$= \int_0^{\pi/3} (1+\tan^2\theta)\sec^2\theta\, d\theta = \int_0^{\pi/3} (\sec^2\theta + \tan^2\theta\sec^2\theta)\, d\theta$

$= \left[\tan\theta + \frac{\tan^3\theta}{3}\right]_0^{\frac{\pi}{3}} = \tan\frac{\pi}{3} + \frac{1}{3}\tan^3\frac{\pi}{3} - \tan 0 - \frac{1}{3}\tan^3 0$

$= \sqrt{3} + \frac{1}{3}(\sqrt{3})^3 - 0 - 0 = 2\sqrt{3}$.

17. The first three nonzero terms of the Maclaurin series of the function $f(x) = \ln(x^2 + x + e)$ are

$f(0) = \ln e = 1$, $f'(x) = \dfrac{2x+1}{x^2+x+e}$, $f'(0) = \dfrac{1}{e}$,

(a) $1 + \frac{x}{e} + \left(\frac{2e-1}{2e^2}\right) x^2$

(b) $1 + \frac{x}{e} + (2e-1)\, x^2$

(c) $1 + \frac{x}{e} + \left(\frac{3e+1}{e^2}\right) x^2$

(d) $1 + x + \frac{1}{e} x^2$

(e) $1 + e^x + \left(\frac{2e+1}{3}\right) x^2$

$$f''(x) = \frac{2(x^2+x+e) - (2x+1)^2}{(x^2+x+e)^2}$$

$$f''(0) = \frac{2e-1}{e^2}$$

The first three nonzero terms of the Maclaurin series of f are.

$$f(0) + \frac{f'(0)}{1!} x + \frac{f''(0)}{2!} x^2$$

$$= 1 + \frac{1}{e} x + \frac{2e-1}{2e^2} x^2.$$

18. The interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n} (x-3)^n$ is

(a) $\left(\frac{5}{2}, \frac{7}{2}\right]$

(b) $\left(\frac{5}{2}, \frac{7}{2}\right)$

(c) $\left[\frac{5}{2}, \frac{7}{2}\right]$

(d) $\left(\frac{5}{2}, \frac{7}{4}\right)$

(e) $\left(\frac{5}{4}, \frac{7}{4}\right]$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{(-2)^{n+1} (x-3)^{n+1}\, n}{(n+1)(-2)^n (x-3)^n} \right|$$

$$= \lim_{n\to\infty} \frac{2n}{n+1} |x-3| = 2|x-3|$$

Using the Ratio Test, the series converges if

$$2|x-3| < 1 \Rightarrow |x-3| < \frac{1}{2} \Rightarrow \frac{5}{2} < x < \frac{7}{2}.$$

$\underline{x = \frac{5}{2}}$: $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n (-\frac{1}{2})^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$ divergent harmonic series.

$\underline{x = \frac{7}{2}}$: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ convergent alternating harmonic series

SO, the interval of convergence is $\left(\frac{5}{2}, \frac{7}{2}\right]$.

19. If $T_2(x)$ is the Taylor polynomial of order 2 generated by the function $f(x) = x^{10}$ at $a = 1$, then $T_2(2) =$

$f(1) = 1$, $f'(x) = 10x^9$, $f'(1) = 10$,

$f''(x) = 90x^8$, $f''(1) = 90$.

$T_2(x) = f(1) + f'(1)(x-1) + \frac{1}{2} f''(1)(x-1)^2$

$= 1 + 10(x-1) + \frac{1}{2}(90)(x-1)^2$

$T_2(2) = 1 + 10(2-1) + 45(2-1)^2$

$= 1 + 10 + 45 = 56$.

(a) 56

(b) 210

(c) 90

(d) 25

(e) 49

20. The series $\sum_{n=1}^{\infty} \frac{1}{n + n(\ln n)^2}$ is

(a) Convergent by the integral test

(b) Divergent by the integral test

(c) Convergent by the ratio test

(d) Convergent since $\lim_{n\to\infty} \frac{1}{n + n(\ln n)^2} = 0$

(e) Divergent by the ratio test

$f(x) = \frac{1}{x + x(\ln x)^2}$

is continuous, positive, decreasing on $[1, \infty)$. (check!)

$\int_1^\infty f(x)dx = \lim_{t\to\infty} \int_1^t \frac{dx}{x(1+(\ln x)^2)}$

$= \lim_{t\to\infty} \int_0^{\ln t} \frac{du}{1+u^2}$, $u = \ln x$, $du = \frac{1}{x}dx$

$= \lim_{t\to\infty} \left[\tan^{-1} u\right]_0^{\ln t}$

$= \lim_{t\to\infty} \left(\tan^{-1}(\ln t) - \tan^{-1} 0\right)$

$= \tan^{-1}(\infty) - 0 = \pi/2$, convergent.

So, the series also is convergent by the integral test.

21. Using the binomial-series, when $|x| < 1/9$, then $\sqrt[3]{1+9x} =$

(a) $1 + 3x - 9x^2 + 45x^3 + \dots$

(b) $1 + 3x + 9x^2 + 27x^3 + \dots$

(c) $1 + 3x - \frac{81}{2}x^2 + \frac{243}{2}x^3 \dots$

(d) $1 - 3x + \frac{27}{2}x^2 + 81x^3 \dots$

(e) $1 + 3x + 9x^2 - 45x^3 + \dots$

$$(1+9x)^{1/3}$$
$$= 1 + \frac{1}{3}(9x) + \frac{\frac{1}{3}(\frac{1}{3}-1)}{2!}(9x)^2 + \frac{\frac{1}{3}(\frac{1}{3}-1)(\frac{1}{3}-2)}{3!}(9x)^3 + \cdots, |9x|<1$$
$$= 1 + 3x + \frac{-2}{9(2)}(9)^2x^2 + \frac{-2(-5)}{27(6)}(9)^3x^3 + \cdots$$
$$= 1 + 3x - 9x^2 + 45x^3 + \cdots$$

22. The sum of the series $-\ln 5 + \frac{(\ln 5)^2}{2!} - \frac{(\ln 5)^3}{3!} + \frac{(\ln 5)^4}{4!} + \dots = S$ is equal to

(a) $\frac{-4}{5}$

(b) $\frac{2}{5}$

(c) $\frac{3}{5}$

(d) $\frac{-3}{5}$

(e) $-\frac{7}{5}$

Since,
$$e^{-x} = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots$$
$$\Rightarrow e^{-\ln 5} = 1 - (\ln 5) + \frac{(\ln 5)^2}{2!} - \frac{(\ln 5)^3}{3!} + \cdots$$
$$= 1 + S$$
Thus, $S = e^{-\ln 5} - 1$
$$= \frac{1}{5} - 1 = -4/5.$$

23. The series $\sum_{n=1}^{\infty} \frac{1}{(\sqrt{3n+1}-1)\sqrt{3n+1}}$ is

(a) Divergent by the limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$

(b) Convergent by the limit comparison test

(c) Divergent by the ratio test

(d) Convergent by the Comparison test

(e) Divergent by the nth term test of divergence.

$$\lim_{n\to\infty} \frac{1}{(\sqrt{3n+1}-1)\sqrt{3n+1}} \Big/ \frac{1}{n}$$

$$= \lim_{n\to\infty} \frac{n}{3n+1-\sqrt{3n+1}}$$

$= \frac{1}{3}$ and $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges

so the series is divergent.

24. The area of the surface obtained by revolving the curve $x = \cosh y$, $0 \le y \le \ln 3$, about the $y-$ axis is (Hint: $\cosh^2 y = \frac{1+\cosh 2y}{2}$)

$\frac{dx}{dy} = \sinh y$, $1+\left(\frac{dy}{dx}\right)^2 = 1+\sinh^2 y$

$= \cosh^2 y$

(a) $\pi\left(\frac{20}{9}+\ln 3\right)$

(b) $\pi\left(\frac{20}{9}-\ln 5\right)$

(c) $\pi\left(\frac{40}{9}+\ln 7\right)$

(d) $\pi\left(\frac{80}{9}+2\ln 3\right)$

(e) $\pi\left(\frac{40}{9}-\ln 5\right)$

$$S = \int_0^{\ln 3} 2\pi x \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dy$$

$$= 2\pi \int_0^{\ln 3} \cosh y \sqrt{\cosh^2 y}\, dy$$

$$= 2\pi \int_0^{\ln 3} \cosh^2 y\, dy = \pi \int_0^{\ln 3} (1+\cosh 2y)\, dy$$

$$= \pi \left[y + \frac{1}{2}\sinh 2y \right]_0^{\ln 3}$$

$$= \pi\left(\ln 3 + \frac{1}{2}\sinh 2\ln 3 - 0 - \frac{1}{2}\sinh 0\right)$$

$$= \pi\left(\ln 3 + \frac{1}{2}\left(\frac{e^{2\ln 3} - e^{-2\ln 3}}{2}\right) - 0\right)$$

$$= \pi\left(\ln 3 + \frac{20}{9}\right).$$

25. The coefficient of x^4 in the Maclaurin series of $\dfrac{e^x}{1-x}$ is

(a) $\frac{65}{24}$

(b) $\frac{8}{3}$

(c) $\frac{61}{34}$

(d) $\frac{31}{12}$

(e) $\frac{5}{3}$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$\frac{e^x}{1-x} = (1 + x + x^2 + x^3 + x^4 + \cdots)\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \cdots\right)$$

coef. of x^4: $\frac{1}{24} + \frac{1}{6} + \frac{1}{2} + 1 + 1$

$$= \frac{5}{24} + \frac{5}{2}$$

$$= \frac{5+60}{24} = \frac{65}{24}.$$

26. $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2} =$ S.

(a) 1

(b) ∞

(c) 2

(d) $\frac{1}{4}$

(e) $\frac{3}{4}$

$$\frac{2n+1}{n^2(n+1)^2} = \frac{n^2 + 2n + 1 - n^2}{n^2(n+1)^2} = \frac{(n+1)^2 - n^2}{n^2(n+1)^2}$$

$$= \frac{1}{n^2} - \frac{1}{(n+1)^2}.$$

so $S_n = \left(\frac{1}{1} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{9}}\right) + \left(\cancel{\frac{1}{9}} - \cancel{\frac{1}{16}}\right) + \cdots$

$$+ \left(\cancel{\frac{1}{(n-1)^2}} - \cancel{\frac{1}{n^2}}\right) + \left(\cancel{\frac{1}{n^2}} - \frac{1}{(n+1)^2}\right).$$

$$= 1 - \frac{1}{(n+1)^2}$$

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(1 - \frac{1}{(n+1)^2}\right) = 1.$$

27. $\int \cos(5x)\,\sin^2(2x)\,dx =$ $\int \cos(5x)\,\frac{1}{2}\left(1-\cos(4x)\right)dx$

$= \frac{1}{2}\int \left(\cos(5x) - \cos(5x)\cos(4x)\right)dx$

$= \frac{1}{2}\int \left(\cos(5x) - \frac{1}{2}\left[\cos(x) + \cos(9x)\right]\right)dx$

$= \frac{1}{10}\sin(5x) - \frac{1}{4}\sin(x) - \frac{1}{36}\sin(9x) + C$

(a) $\dfrac{1}{10}\sin(5x) - \dfrac{1}{4}\sin(x) - \dfrac{1}{36}\sin(9x) + c$

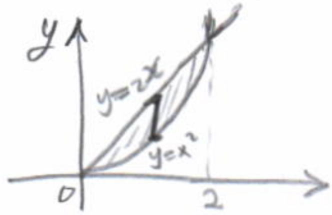
(b) $-\dfrac{1}{4}\cos(x) - \dfrac{1}{36}\sin(9x) + c$

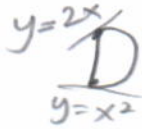
(c) $\dfrac{1}{10}\cos(5x) - \dfrac{1}{4}\cos(x) + c$

(d) $\dfrac{1}{36}\sin(9x) + \dfrac{1}{10}\cos(5x) + \dfrac{1}{4}\cos(x) + c$

(e) $\dfrac{1}{10}\cos(5x) + \dfrac{1}{36}\cos(9x) + c$

28. The base of a solid is the region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant. If the cross-sections perpendicular to the x-axis are semicircles with diameters running across the base of the solid, then the volume of the solid is given by





$D = 2x - x^2$

$r = \frac{D}{2}$

$A(x) = \frac{1}{2}\pi r^2 = \frac{\pi}{2}\left(\frac{2x-x^2}{2}\right)^2$

$= \frac{\pi}{2}\,\frac{4x^2-4x^3+x^4}{4}$

$x^2 = 2x \Rightarrow x = 0,\ x = 2.$

$V = \int_0^2 A(x)\,dx = \frac{\pi}{2}\int_0^2 \left(x^2 - x^3 + \frac{x^4}{4}\right)dx.$

(a) $\dfrac{\pi}{2}\displaystyle\int_0^2 \left(\frac{x^4}{4} - x^3 + x^2\right)dx$

(b) $\pi \displaystyle\int_0^2 \left(x^4 - 4x^3 + x^2\right)dx$

(c) $\pi \displaystyle\int_0^2 \left(4x^2 - x^4\right)dx$

(d) $\dfrac{\pi}{2}\displaystyle\int_0^2 \left(x^2 - 2x\right)^2 dx$

(e) $\pi \displaystyle\int_0^2 (x^5 - x^4 + x^2)\,dx$