Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals

AS483: Actuarial Risk Theory & Credibility
Dr. Mohammad H. Omar
Final Exam Term 142 FORM A
Monday May 18, 2015
8.00am-10.30am

Name______________________________ ID#:__________ Serial #:___

Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the **cheating rules** of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financail calculators only. Write important steps to arrive at the solution of the following problems.

The test is 150 minutes, GOOD LUCK, and you may begin now!

Question	Total Marks	Marks Obtained	Comments
1	6+2=8		
2	3*4+3=15		
3	5		
4	3+4=7		
5	3*3+1=10		
6	3+4=7		
7	4+3+4+3=14		
8	4+5=9		
9	3+3+4=10		
10	4+1=5		
Total	90		

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1. (6+2=8 points) Let Θ_1 represent the risk factor for claim numbers and let Θ_2 represent the risk factor for the claim severity for a line of insurance. Suppose that Θ_1 and Θ_2 are independent. Suppose also that, given $\Theta_1 = \theta_1$, the claim number N is Poisson distributed and, given $\Theta_2 = \theta_2$, the severity Y is exponentially distributed. The expectations of the hypothetical means and process variances for the claim number and severity as well as the variance of the hypothetical means for frequency are repectively,

$$\mu_N = 0.1, \qquad \upsilon_N = 0.1, \qquad a_N = 0.05,$$
$$\mu_Y = 100, \qquad \upsilon_Y = 25000$$

Three observations are made on a particular policyholder and we observe **total claims** of 200.

a) Determine the **Biihlmann credibility factor** for this policyholder.

b) Determine the **Biihlmann premium** for this policyholder.

2. (3*4+3=15 points) Past data on a portfolio of group policyholders are given in the Table below. Estimate the **Biilhmann-Straub credibility premiums** to be charged to **each** group in year 4.

Policyholder		**Year**			
		1	**2**	**3**	**4**
Claims	1	-	20000	25000	-
No. in group		-	100	120	110
Claims	2	19000	18000	17000	-
No. in group		90	75	70	60
Claims	3	26000	30000	35000	-
No. in group		150	175	180	200

3. (5 points) A group of insureds had 6000 claims and a total loss of 15 600 000. The prior estimate of the total loss was 16 500 000. Using a standard for full credibility that calls for a minimum of 19542.51 claims, determine the limited flunctuation credibility **estimate** of the total loss for the group.

4. (3+4=7 points) A group of 340 insureds in an high-crime area submit the 210 theft claims in a **one year** period as given in the Table below. Each insured is assumed to have a *Poisson* distribution for the number of thefts, but the mean of such a distribution may vary from one insured to another. If a particular insured experienced *two claims* in the observation period, determine the **Biihlmann credibility estimate** for the number of claims for this insured in the next period.

No. of Claims	No. of Insureds
0	200
1	80
2	50
3	10

5. (3*3+1=10 points) At any time a member of a *pension* plan is in one of **three** states: (1) *employed* (e), (2) *alive* but *no longer employed* (n), or (3) *dead* (d). Let q^{ab} denote the probability that a current or former member of the plan is in state b at the **end of a year** given that the member was in state a at the **beginning of that year**. The probabilities are **constant over time** and **independent** of age. They are: $q^{ee} = 0.90$, $q^{en} = 0.08$, $q^{ed} = 0.02$, $q^{nn} = 0.95$, $q^{nd} = 0.05$, and $q^{dd} = 1.00$. Any probabilities not listed are zero.

   At the beginning of a particular year there are 200 members, all of whom are *employed*. Using the conditional approach for multinomial categories, **simulate** the number of members in **each** of the three states **two years from now**. Use the uniform random numbers 0.123, 0.876, 0.295, 0.623, and 0.426.

6. (3+4=7 points) You are simulating observations from an exponential distribution with $\theta = 100$.

a) How many simulations are needed to be 90% certain of being within 2% of each of the mean?
b) How many simulations are needed to be 90% certain of being within 2% of the probability of being below 200?

7. (4+3+4+3=14 points) An automobile insurance policy provides benefits for accidents caused by both underinsured and uninsured motorists. Data on 1000 policies revealed the total number of underinsured and uninsured claims for each of the 1000 policies and the results are as in Table 14.11 below.

No. of Claims (underinsured and uninsured)	**No. of policies**
0	861
1	121
2	13
3	3
4	1
5	0
6	1
7+	0

(a) For a Poisson model of the Total number of claims,.determine the **maximum likelihood estimator** (mle) of λ.

(b) Determine the **mle** of β for a geometric model.

(c) Determine the **method of moments** estimate of r and β for a negative binomial model.

(d) Assume that $m = 7$. Determine the **mle** of q of the binomial model.

8. (4+5=9 points) For the Weilbull (100,1/2) distribution, obtain the following risk measures:

   a) VaR at the 99.9% security level

   b) TVaR at the 99.9% security level.

9. (3+3+4=10 points) A ground-up model of individual losses has the gamma distribution with parameters $\alpha = 2$ and $\theta = 80$. The number of losses has the negative binomial distribution with $r = 2$ and $\beta = 3$. An ordinary deductible of 50 and a loss limit of 175 (before imposition of the deductible) are applied to each individual loss.

a) Determine the distribution of the number of payments on a *per-loss* basis.

b) Determine the cumulative distribution function of the amount Y^P of a payment *given* that a payment is made.

c) Discretize the severity distribution from (b) using the *method of rounding* and a span of 40.

10. (1+4=5 points) You are given a sample of two values, 5 and 9. You estimate Var(X) using the estimator $g(X_1, X_2) = \frac{1}{2}\sum(X_i - \overline{X})^2$ Determine the bootstrap approximation to the mean square error (MSE) of g.

A) 1
B) 2
C) 4
D) 8
E) 16

Final answer (1 point)
Work shown (4 points)

The answer is ____

END OF TEST PAPER

9. (6+4=10 points) For a policy that covers both fire and wind losses, you are given that a sample of fire losses was 3 and 4 and a sample of wind losses for the same period was 0 and 3. Fire and wind losses are independent and do not have identical distributions. Based on the sample, you estimate that adding a deductible of 2 per wind claim will eliminate 20% of total losses. Determine the **bootstrap approximation** to the MSE of the estimate.

Solution: see *KPW Chapter 20 Q 20.24 pg 458*

There are 4 possible bootstrap samples for the 2 fire losses and 4 for wind losses, making 16 equally likely outcomes. There are 9 unique cases, as follows. The losses are presented as the first fire loss, second fire loss, first wind loss, and second wind loss.

Case 1 : loss is (3, 3, 0, 0); total is 6; eliminated is 0; fraction is 0; square error is $(0 - 0.2)^2 = 0.04$; probability 1/16.

Case 2 : loss is (3, 3, 0, 3); total is 9; eliminated is 2; fraction is 0.22; error is 0.0005; probability 2/16 [includes (3, 3, 3, 0)].

Case 3 : loss is (3, 3, 3, 3); total is 12; eliminated is 4; fraction is 0.33; error is 0.0178; probability 1/16.

Case 4 : loss is (3, 4, 0, 0); total is 7; eliminated is 0; fraction is 0; error is 0.04; probability 2/16.

Case 5 : loss is (3, 4, 0, 3); total is 10; eliminated is 2; fraction is 0.2; error is 0; probability 4/16.

Case 6 : loss is (3, 4, 3, 3); total is 13; eliminated is 4; fraction is 0.3077; error is 0.0116; probability 2/16.

Case 7 : loss is (4, 4, 0, 0); total is 8; eliminated is 0; fraction is 0; error is 0.04; probability 1/16.

Case 8 : loss is (4, 3, 0, 3); total is 11; eliminated is 2; fraction is 0.1818; error is 0.0003; probability 2/16.

Case 9 : loss is (4, 4, 3, 3); total is 14; eliminated is 4; fraction is 0.2857; error is 0.0073; probability 1.16.

The MSE is $[1(0.04) + 2(0.0005) + ... + 1(0.0073)]/16 = 0.0131$.