Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals

AS483: Actuarial Risk Theory & Credibility

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Major 2 Exam   Term 142   FORM A
Wednesday Apr 22, 2015
7.00pm-8.30pm

Name______________________________ ID#:__________ Serial #:___

Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the **cheating rules** of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financail calculators only. Write important steps to arrive at the solution of the following problems.

The test is 90 minutes, GOOD LUCK, and you may begin now!

Question	Total Marks	Marks Obtained	Comments
1	5		
2	8		
3	6+4+3+3+4=20		
4	3+4=7		
5	4+3+3=10		
6	6+4=10		
Total	60		

Extra blank page

1. (5 points) Determine the **loss elimination ratio** for the distribution given below with an ordinary deductible of 4000.

$$F_X(x) = \begin{cases} 0, & x < 0, \\ 1 - 0.3e^{-0.00001x}, & x \geq 0. \end{cases}$$

2. (8 points) A compound distribution has a **zero-modified** binomial distribution with $m = 3$, $q = 0.3$, and $p_0^M = 0.4$. Individual payments are 0, 50, and 150 with probabilities 0.3, 0.5, and 0.2, respectively. Use the recursive formula to determine the probability distribution of S (Hint: You need to find $f_S(s)$ for $s = 0, 1, 2, 3, 4$ only).

3. (6+3+3+4+4=20 points) A ground-up model of individual losses has the gamma distribution with parameters $\alpha = 2$ and $\theta = 100$. The number of losses has the negative binomial distribution with $r = 2$ and $\beta = 1.5$. An ordinary deductible of 50 and a loss limit of 175 (before imposition of the deductible) are applied to each individual loss.

a) Determine the **mean and variance** of the aggregate payments on a *per-loss* basis.
b) Determine the **distribution** of the number of payments.
c) Determine the **cumulative distribution function** of the amount Y^P of a payment given that a payment is made.
d) **Discretize** the *severity* distribution from (c) using the *method of rounding* and a *span* of 40.
e) Use the **recursive** formula to calculate the discretized distribution of **aggregate payment** up to a *discretized* amount paid of 120.

4. (3+4=7 points) Consider a *group life* insurance contract with an *accident death benefit.* Assume that for all members the probability of death in the next year is 0.02 and that 30% of the deaths are accidental. For 50 employees, the benefit for an ordinary death is 50000 and for an accidental death it is 100000. For the remaining 25 employees, the benefits are 75000 and 150000, respectively. Develop an individual risk model and determine its **mean** and **variance**.

5. (4+3+3=10 points) Let $x_1, ..., x_n$ be a random sample from a population with pdf $f(x) = \theta^{-1}e^{-x/\theta}$, $x > 0$. This exponential distribution has a mean of θ and a variance of θ^2. Consider the sample mean, $\bar{X}$, as an estimator of θ. It turns out that $\bar{X}/\theta$ has a gamma distribution with $\alpha = n$ and $\theta = 1/n$, where in the second expression the "θ" on the left is the parameter of the gamma distribution. For a sample of size 50 and a sample mean of 550, develop 95% confidence intervals of θ by each of the following methods. In each case, if the formula requires the true value of θ, substitute the estimated value.

a) Use the gamma distribution to determine an exact interval estimate of θ.

b) First estimating the variance prior to solving the inequalities and then use a normal approximation to determine the interval estimate of θ.

c) Without estimating the variance prior to solving the inequalities, use a normal approximation to determine the interval estimate of θ.

6. (6+4=10 points) An automobile insurance policy provides benefits for accidents caused by both underinsured and uninsured motorists. Data on 1000 policies revealed the information recorded in the table below.

No of claims	Underinsured	Uninsured
0	901	947
1	92	50
2	5	2
3	1	1
4	1	0
5^{+} (5 or more)	0	0

a) Determine the mle of λ_1 for a poisson model of N_1=number of underinsured claims and the mle of λ_2 for a poisson model for N_2=number of uninsured claims..

b) Assuming that N_1 and N_2 are independent events. Determine a suitable model for $N = N_1 + N_2$ and the mle of its parameter.

END OF TEST PAPER