Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals

AS475: Survival Models for Actuaries

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Final Exam Term 142 FORM A
Thursday May 21 2015
8.00am-10.30am

Name__________________________________ ID#:__________ Serial #:___

Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the **cheating rules** of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financail calculators only. Write important steps to arrive at the solution of the following problems.

The test is 150 minutes, GOOD LUCK, and you may begin now!

Question	Total Marks	Marks Obtained	Comments
1	6+2=8		
2	6		
3	8		
4	8		
5	6+2=8		
6	2+2+4+8=16		
7	6+3=9		
8	3+4=7		
9	1+4=5		
Total	75		

Extra blank page

1. (6+2=8 points) Twenty claim amounts were sampled from a Pareto distribution with $\alpha = 2$ and θ unknown. The Pareto pdf is as follows:

$$f(x) = \frac{\alpha\theta^a}{(x+\theta)^{a+1}}$$

The maximum likelihood estimate of θ is 7.0. Also, $\sum_{j=1}^{20} \ln(x_j + 7.0) = 49.01$ and $\sum_{j=1}^{20} \ln(x_j + 3.1) = 39.30$. The likelihood ratio test is used to test the null hypothesis that $\theta = 3.1$.

a) Determine the p**-value** for this test. (Hint: set this up and use EXCEL CHISQ.DIST.RT function to evaluate it)

b) Provide your conclusion for this test.

2. (6 points) A random sample of 3 losses is 1.0 1.0 2.5. If the assumed distribution is uniform on [0,4], calculate A^2, the Anderson-Darling statistic for this fit.

Direction: For questions 3 and 4 below, consider the following data set.

Suppose that Alice (A), Salis (S), and Calis (C) are the only three subjects in the dataset shown below. All three subjects have two *recurrent* events that occur at different times.

ID	Status	Stratum	Start	Stop	tx
A	1	1	0	70	1
A	1	2	70	90	1
S	1	1	0	20	0
S	1	2	20	30	0
C	1	1	0	10	1
C	1	2	10	40	1

3. (8 points) Fill in the following data layout describing survival (in weeks) **to the first event (stratum 1)**. Recall that m_f and q_f denote the number of failures and censored observations at time $t_{(f)}$. The survival probabilities in the last column use the KM product limit formula.

$t_{(f)}$	n_f	m_f	q_f	$R(t_{(f)})$	$S_1(t_{(f)})$
0	3	0	0	{A,S,C}	1.00
10	-	-	-	-	-
-	-	-	-	-	-
-	-	-	-	-	-

4. (8 points) Fill in the following data layout describing survival (in weeks) **to the first to second event (stratum 2)** using the **Gap Time** approach.

$t_{(f)}$	n_f	m_f	q_f	$R(t_{(f)})$	$S_1(t_{(f)})$
0	3	0	0	{A,S,C}	1.00
10	-	-	-	-	-
-	-	-	-	-	-
-	-	-	-	-	-

5. (6+2=8 points) The addicts.dat dataset are analyzed with two models as follows:

Model A
Weibull regression
accelerated failure-time form

Log-likelihood =-260.74854

_t	*Coef.*	*Std.Err*	*z*	$P>\|z\|$
clinic	0.698	.158	4.42	0.000
prison	0.145	.558	0.26	0.795
dose	0.027	.006	4.60	0.000
pris.dose	-0.006	.009	-0.69	0.492
_const	3.977	.376	10.58	0.000
/ln_p	0.315	.068	4.67	0.000
p	1.370467			
1/p	0.729678			

Model B
Weibull regression
accelerated failure-time form
Gamma frailty

Log-likelihood =-260.74854

_t	*Coef.*	*Std.Err*	*z*	$P>\|z\|$
clinic	0.698	.158	4.42	0.000
prison	0.145	.558	0.26	0.795
dose	0.027	.006	4.60	0.000
pris.dose	-0.006	.009	-0.69	0.492
_const	3.977	.376	10.58	0.000
/ln_p	0.315	.068	4.67	0.000
p	1.370467			
1/p	0.729678			
theta	.00000002		.0000262	

LRT of theta=0;
chibar2(01)=0.00
Prob>=chibar2=1.00

a) Using Model A, estimate the acceleration factor with a 95% confidence interval comparing CLINIC=2 vs CLINIC=1. Interpret this result.

b) Did the addition of the Gamma frailty component change the estimates for other parameters (other than θ) and the log likelihood compared to that in the original model A?

c) What is the result of the likelihood ratio test for the hypothesis H_0: $\theta = 0$..What does this result mean?

6. (2+2+4+8=16 points) Consider a study of the effect of a bone marrow transplant for leukemia on leukimia-free survival, where two transplant failures types can occur: *relapse of leukimia* and *nonrelapse death* (without prior relapse of leukimia). Suppose that in *hospital A*, 100 first patient undergo such a transplant and that within the first 4 years after transplant, 60 die *without relapse* by year 2 and 20 *relapses* during year 4. Suppose that in *hospital B*, 100 patients undergo such a transplant but after transplant, there are 20 *non-relapse* deaths by year 1, 15 *relapses* during year 2, 40 *non-relapse* deaths between year 3 and 4, and 5 *relapses* during year 4.

(a) What are the *competing risks* in this study?

(b) What is the *proportion* of initial patients in hospital A and B, respectively, that have *leukimia relapse* by 4 years?

(c) Complete the missing information in the following tables providing the Kaplan-Meier (KM) curves for *relapse of leukimia* for each study. How have both tables treated the competing risk for non-relapse death in calculation of the KM probabilities?

Hospital A

t_j	n_j	m_j	q_j	$S(t_j)$
0	100	0	60	1
2	40	0	0	1
4			20	

Hospital B

t_j	n_j	m_j	q_j	$S(t_j)$
0	100	0	20	1
1	80	0	0	1
2	80	15	0	0.8125
3	65	0	40	0.8125
4			20	

(d) Compute the *CIC curves* for each hospital by completing the missing information in the following tables.

Hospital A

t_f	n_f	m_f	$\hat{h}_{ca}(t_f)$	$\hat{S}(t_{f-1})$	$\widehat{I}_{ca}(t_f)$	$CIC(t_f)$
0	100	0	0	-	-	-
2	40	0	0	1	0	0
4	40	20				

Hospital B

t_f	n_f	m_f	$\hat{h}_{ca}(t_f)$	$\hat{S}(t_{f-1})$	$\widehat{I}_{ca}(t_f)$	$CIC(t_f)$
0	100	0	0	-	-	-
1	80	0	0	1	0	0
2	80	15				
3	65	0				
4	25	5				

7. (6+3=9 points) Suppose that the following remission durations are observed from 10 patients ($n = 10$) with solid tumors.

Six patients relapse at 3.0, 6.0, 6.0, 10, 12, and 15 months;
1 patient is lost to follow-up at 8.4 months; and
3 patients are still in remission at the end of the study after 3.5, 5.2, and 10 months.
(a) Determine the survivorship function $\widehat{S}(t)$ and

(b) Estimate (by linear interpolation) the median remission time

8. (3+4=7 points) A computer analysis on leukimia patients ($n = 42$) shows the following results:

Variable name	Coef.	Std. Err.	$p > \|z\|$	Haz. Ratio .	[95% Conf	interval]
$Log\ WBC$	1.170	0.499	0.019	3.222	1.213	8.562
Rx	0.267	0.566	0.637	1.306	0.431	3.959
$Sex \times Log\ WBC$	0.469	0.720	0.515	1.598	0.390	6.549
$Sex \times Rx$	1.592	0.923	0.084	4.915	0.805	30.003

No. of subjects = 42 Log likelihood = −55.835 Stratified by *Sex*

(a) Write the Cox model represented by the computer printout above.

(b) If you know that a reduced model with only $Log\ WBC$ and Rx as predictors results in a log likehood of −57.560, at a 5% significance level, what can you conclude about the **full** model?

9. (4+1=5 points) You are given:

(i) A sample of insurance claim payments is:

29 64 90 135 182

(ii) Claim sizes are assumed to follow an exponential distribution.

(iii) The mean of the exponential distribution is estimated using the *method of moments.*

Calculate the value of the **Kolmogorov-Smirnov** test statistic.

a) 0.14

b) 0.16

c) 0.19

d) 0.25

e) 0.27

Final answer (1 point)

Work shown (4 points)

So the answer is ____

END OF TEST PAPER