Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals

AS475: Survival Models for Actuaries
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Major 1 Exam Term 142 FORM A **SOLUTION**
Thursday March 5 2014
12.30pm-1.45pm

Name_____ ID#:_____ Serial #:___

Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the **cheating rules** of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financail calculators only. Write important steps to arrive at the solution of the following problems.

The test is 90 minutes, GOOD LUCK, and you may begin now!

Question	Total Marks	Marks Obtained	Comments
1	8+4=12		
2	3+3=6		
3	4+4+7=15		
4	3+3+3=9		
5	4+1=5		
6	4+3+3+3=13		
Total	60		

Extra blank page

1. (8+4=12 points) The data in Table 11.8 below are from loss distributions, representing the total damage done by 35 hurricanes between years 1949 and 1980. The losses have been adjusted for inflation to be in 1981 dollars. The entries represent all hurricanes for which the trended loss was in excess of 5000000 (*i.e.*, 5×10^6). The federal government is considering funding a program that would provide 100% payment for all damages for any hurricane causing damage in excess of 5000000. You have been asked to make some preliminary estimates.

Table 11.8 Trended Hurricane Losses.

$Year$	$Loss(10^6)$	$Year$	$Loss(10^6)$	$Year$	$Loss(10^6)$
1964	6.766	1964	40.596	1975	192.013
1968	7.123	1949	41.409	1972	198.446
1971	10.562	1959	47.905	1964	227.338
1956	14.474	1950	49.397	1960	329.511
1961	15.351	1954	52.600	1961	361.200
1966	16.983	1973	59.917	1969	421.680
1955	18.383	1980	63.123	1954	513.586
1958	19.030	1964	77.809	1954	545.778
1974	25.304	1955	102.942	1970	750.389
1959	29.112	1967	103.217	1979	863.881
1971	30.146	1957	123.680	1965	1638.000
1976	33.727	1979	140.136		

Note that $\sum_{j=1}^{35} x_j = 7171.514$, $\sum_{j=1}^{35} x_j^2 = 5184706.28428$, $\sum_{j=1}^{35} x_j^3 = 5953050770.49997$,

a) Estimate the mean, standard deviation, coefficient of variation, and skewness for the population of hurricane losses.

b) Consider an insurance policy that pays the following payments for **limited** loss

$$Y = \begin{cases} X & X < u \\ u & X \geq u \end{cases} .$$

Estimate the first and second **limited moments** at a limit of $u = 500 \times 10^6$. (Hint: you can think of observations above u as right censored)

2. (3+3=6 points) The following 20 windstorm losses (in millions of dollars) were recorded in one year:

1	1	1	1	1	2	2	3	3	4	6	6	8	10	13	14	15	18	22	25

(a) Construct an ogive using class boundaries at 0.5, 3.5, 8.5, 15.5, and 29.5.

(b) Construct a histogram using the same boundaries as in part (a).

3. (4+4+7=17 points) For Data set D2 below with 40 insurance policyholders, policy number (i), time first observed (d_i), and time of last observation (x_i=death time,u_i=surrender time) were recorded.

i	d_i	x_i	u_i	i	d_i	x_i	u_i	i	d_i	x_i	u_i
1	0		0.1	11	0	2.9		31	0.3		5
2	0		0.5	12	0		3.9	32	0.7		5
3	0		0.8	13	0	4		33	1	4.1	
4	0	0.8		14	0		4	34	1.8	3.1	
5	0		1.8	15	0		4.1	35	2.1		3.9
6	0		1.8	16	0	4.8		36	2.9		5
7	0		2.1	17	0		4.8	37	2.9		4.8
8	0		2.5	18	0		4.8	38	3.2	4	
9	0		2.8	$19-30$	0		5	39	3.4		5
10	0	2.9						40	3.9		5

(a) Complete the following table for the risk set calculations for time to surrender.

j	y_j	s_j	r_j	j	y_j	s_j	r_j
1	0.1	1	$0+30-0-0=30$	7	2.8	1	$28+1-1-0=27$
2	0.5	1	$30+1-1-0=30$	8	3.9	2	27
3	0.8	1		9	4.0	1	26
4	1.8	2	$30+1-1-1=29$	10	4.1	1	
5	2.1	1		11	4.8	3	21
6	2.5	1	28	12	5.0	17	

(b) Determine the **Kaplan-Meier estimate** for the time to surrender for this Data Set D2.

(c) Determine the **Nelson-Aalen estimate** of $H(t)$ and $S(t)$ for Data Set D2 where the variable is time to surrender.

4. (3+3+3=9 points) A mortality study is based on the period January 1 2009 through Dec 31 2011. The company assigned insuring ages by age on last birthday. Five policies were observed with the following records.

Born on	Purchase policy on	last observed	event	New Assigned birthday	*age* 1^{st} *obs*	*age last obs*
4-1976	8-2008	1-2012	active policyholder	8-1976	32-5	35-5
6-1976	7-2008	9-2010	died		32-6	34-2
8-1976	2-2010	2-2011	surrendered	2-1977	33-0	
5-1976	6-2009	3-2010	died	6-1976	33-0	33-9
7-1976	3-2009	5-2011	surrendered	3-1977		34-2

a) Complete the blank information above

b) Complete the exact exposures below.

Born on	Purchase policy on	last observed	event	*age* 1^{st} *obs*	*age last obs*	32-33	33-34	34-35	35-36
4-1976	8-2008	1-2012	active policyholder	$32-5$	$35-5$	7	12	12	5
6-1976	7-2008	9-2010	died	$32-6$	$34-2$		12	2	0
8-1976	2-2010	2-2011	surrendered	$33-0$	$\mathbf{34-0}$	0		0	0
5-1976	6-2009	3-2010	died	$33-0$	$33-9$	0	9	0	
7-1976	3-2009	5-2011	surrendered	$\mathbf{32-0}$	$34-2$	12		2	0
							57	16	

c) Compute $\widehat{q}_{33}$ and $\widehat{q}_{34}$.

5. (4+1=5 points) From a population having distribution function F, you are given the following sample:

4.0, 6.6, 6.6, 8.0, 8.0, 9.4, 9.4, 9.4

Calculate the kernel density estimate of $F(8)$, using the uniform kernel with bandwidth 2.8.

a) 0.63
b) 0.53
c) 0.50
d) 0.41
e) 0.31

Work Shown (4 points):

The answer is .

6. (4+3+3+3=13 points) Consider the failure data below and the hypothesis of equivalent survival curves,

		# failures		**# in risk set**		**# expected**		**Observed - expected**	
j	$t_{(j)}$	$m_{(1j)}$	$m_{(2j)}$	$n_{(1j)}$	$n_{(2j)}$	$e_{(1j)}$	$e_{(2j)}$	$m_{(1j)} - e_{(1j)}$	$m_{(2j)} - e_{(2j)}$
1	1	0	2	21	21	1	1	-1.00	1.00
2	2	0	2	21	19	1		-1.05	1.05
3	3	0	1	21	17	21/38	17/38		0.55
4	4	0	2	21	16	42/37	32/37	-1.14	1.14
5	5	0	2	21	14	42/35		-1.20	
6	6	3	0	21	12	21/11	12/11	1.09	-1.09
7	7	1	0	17	12	17/29	12/29	0.41	-0.41
8	8	0	4	16	12	16/7	12/7	-2.29	2.29
9	10	1	0	15	8		8/23		-0.35
10	11	0	2	13	8	13/7	16/21	-1.24	1.24
11	12	0	2	12	6	12/9		-1.33	
12	13	1	0	12	4	3/4	1/4	0.25	-0.25
13	15	0	1	11	4	11/15	4/15	-0.73	0.73
14	16	1	0	11	3	11/14	3/14	0.21	-0.21
15	17	0	1	10	3	10/13	3/13	-0.77	0.77
16	22	1	1	7	2	14/9	4/9	-0.56	0.56
17	23	1	1	6	1	12/7	2/7	-0.71	0.71
Totals		9	21			19.26	10.74		

(a) complete the blank spaces (provide sample calculation below)

(b) calculate the approximate chi-square statistics.

(c) calculate the log rank statistic. (Hint: $\widehat{Var}(O_2 - E_2) = 6.2570$)

(d) decide at $\alpha = 0.10$, if the two groups have equal survival curves

. END OF TEST PAPER