KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
DHAHRAN, SAUDI ARABIA

STAT 319: Probability & Statistics for Engineers & Scientists
Semester 141
Third Major Exam
Wednesday December 17, 2014
5:00 – 6:30 pm

Please circle your instructor name:

Abbas Al-Sabah Al-Sawi

Anabosi Malik

Saleh

Name: KEY ID #: 0000 Section #: ALL Serial #:

Question No	Full Marks	Marks Obtained
1	**14**	
2	**9**	
3	**9**	
4	**10**	
5	**8**	
Total	**50**	

Q1. (6+3+5 marks) An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed with a standard deviation of 40 hours.

a. If a sample of 25 bulbs has an average life of 780 hours, find a 96% confidence interval for the mean life length of all bulbs produced by this firm, stating the assumptions used to construct the interval.

Let X: Life length , $\sigma = 40$, $n = 25$, $\bar{x} = 780$

$z_{\frac{\alpha}{2}} = z_{\frac{0.04}{2}} = z_{0.02} = 2.05$ 1

A 96% C.I. for μ is $\bar{x} \pm z_{0.02} \frac{\sigma}{\sqrt{n}}$ 1

$= 780 \pm (2.05) \frac{40}{\sqrt{25}}$ 1 $= 780 \pm 16.40 = \boxed{[763.6, 796.4]}$

Assumptions used:

3 ① Small sample ② Population is Normal ③ σ is known.

b. How large a sample is needed if we wish to be 96% confident that our sample mean will be within 10 hours of the true mean?

1

$n \geq \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{(2.05)(40)}{10}\right)^2 = 67.24$ 1

$\boxed{n \approx 68}$ 1

c. Do you agree with the belief that the mean life length of all bulbs is not 795 hours? Use 4% level of significance.

① $H_0: \mu = 795$ vs. $H_1: \mu \neq 795$, $\alpha = 0.04$

② A 96% C.I. for $\mu = [763.6, 796.4]$ from (a).

③ If $\mu_0 \notin$ C.I. ⇒ Reject H_0.

④ Since $795 \in$ C.I. ⇒ Do NOT reject H_0

⑤ Therefore, NO, I do NOT agree.

1 each

Q2. (~~6~~ 7+2 ~~+3~~ marks) In a study of water contamination in wells, 223 wells, divided into 103 private wells and 120 public wells, were tested and the results are tabulated below.

	Type of Well		
Pollutant Level	Private	Public	Total
Low	**81**	**72**	153
High	**22**	**48**	70
Total	103	120	223
	n_1	n_2	

a. A resident claimed that more public wells are highly polluted than private wells. Statistically, by using the *p*-value approach to testing, do you agree?

Let p_1 = Proportion of highly polluted Private wells.
p_2 = " " " " Public " .

$\hat{p}_1 = \frac{22}{103} = 0.2136$, $\hat{p}_2 = \frac{48}{120} = 0.4$, $\boxed{\alpha = 0.05}$ (assumed)

$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{22+48}{103+120} = \frac{70}{223} = \boxed{0.3139}$ 1

① $H_0: p_1 = p_2$ vs. $H_1: p_1 < p_2$ 1

② $Z_0 = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \dfrac{0.2136 - 0.4}{\sqrt{(0.3139)(0.6861)\left(\frac{1}{103} + \frac{1}{120}\right)}} = \boxed{-2.99}$ 1

③ p-Value $= P(Z < Z_0) \overset{1}{=} P(Z < -2.99) = \boxed{0.0014}$ 1

④ If p-value $< \alpha \Rightarrow$ Reject H_0.

⑤ Since $0.0014 < 0.05 \Rightarrow$ Reject H_0. 1

⑥ Therefore, YES, I agree. 1

b. Check the assumptions of your test in (a)?

2
$n_1\hat{p}_1 = x_1 = 22 \geq 5$ ✓
$n_1\hat{q}_1 = n_1 - x_1 = 81 \geq 5$ ✓
$n_2\hat{p}_2 = x_2 = 48 \geq 5$ ✓
$n_2\hat{q}_2 = n_2 - x_2 = 72 \geq 5$ ✓

Q3. (6+3 marks) A Videocassette recorder (VCR) tape manufacturer is experimenting with a new technology, which hopefully will produce longer-lasting tapes. Thirty of the old-style tapes and thirty utilizing the new technology were used in an experiment. The number of re-recordings is assumed to be normally distributed. The number of re-recordings were observed and shown in the table below.

	Old-Style Tapes	**New Technology Tapes**
No. of accepted tapes	18	23
n	30	30

a. Using the critical value approach to testing, do the data allow us to infer, at the 2% significance level, that the proportion of unacceptable old tapes exceeds 25%?

$n = 30$, $x = 12 \Rightarrow \hat{p} = \frac{x}{n} = \frac{12}{30} = 0.4$, $\alpha = 0.02$

① $H_0: p = 0.25$ vs. $H_1: p > 0.25$ ①

② Assumptions: $np_0 = \frac{30}{4} = 7.5 \geq 5$ ✓ ①

$nq_0 = 30 \times \frac{3}{4} = 22.5 \geq 5$ ✓

③ $Z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.4 - 0.25}{\sqrt{\frac{(0.25)(0.75)}{30}}} = 1.897$ ①

④ $Z_\alpha = Z_{0.02} = 2.05$

⑤ If $Z_0 > Z_\alpha \Rightarrow$ Reject H_0. ①

① ⑥ Since $1.897 \not> 2.05 \Rightarrow$ Do NOT reject H_0, hence, NO ① the data do NOT allow to infer that $p > 0.25$.

b. What is the maximum sample size that can give a sample proportion within 0.1 from the population proportion of acceptable old tapes with a 98% confidence level?

The maximum sample size $= n \leq \left(\frac{Z_{\alpha/2}}{E}\right)^2 \frac{1}{4}$ ①

Since $\alpha = 0.02 \Rightarrow Z_{\frac{\alpha}{2}} = Z_{0.01} = 2.33$ ① $\Rightarrow$

$n \leq \left(\frac{2.33}{0.1}\right)^2 \frac{1}{4} = 135.7225 \Rightarrow$

$\boxed{n = 135}$ ① Since we can not go less than 135.

Q4. (8+2 marks) A new purification unit is installed in a chemical process. Before its installation, a random sample yielded the following data about the percentage of impurity: $n_1 = 40$, $\overline{x_1} = 9.85$, $s_1^2 = 6.79$. After installation, another random sample resulted in $n_2 = 26$, $\overline{x_2} = 8.18$, $s_2^2 = 6.18$.

a. Using the *p*-value approach to testing, can you conclude that the new purification device has reduced the mean percentage of impurity?

$\alpha = 0.05$ (assumed)

(1) $H_0: \mu_1 = \mu_2$ vs. $H_1: \mu_1 > \mu_2$ $(\mu_1 - \mu_2 > 0)$ (1)

(2) $$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \delta_0}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{(9.85 - 8.18) - 0}{2.5596\sqrt{\frac{1}{40}+\frac{1}{26}}} = \boxed{2.59}$$ (1)

$$s_p = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{39(6.79)+25(6.18)}{40+26-2}} = \sqrt{6.55172} = \boxed{2.5596}$$ (1)

(3) p-value $= P(t_{64} > t_0)$ (1)

$= P(t_{64} > 2.59) \Rightarrow$

$\boxed{0.005 < \text{p-value} < 0.01}$ (1)

(4) If p-value $< \alpha \Rightarrow$ Reject H_0. (1)

(5) since p-value $< 0.01 \ll 0.05 \Rightarrow$ Reject H_0. (1)

(6) Therefore, YES, we can (1) conclude that the new device has reduced the impurity, at 5% level of significance.

b. What assumption(s) did you **need** in solving the problem in part (a)?

(1) We need to assume that the impurity distribution of both devices is Normal.

(2) We, also, assumed the unknown σ's to be EQUAL.

(1) each

Q5. (6+2 marks) A civil engineer wants to compare two instruments for measuring some chemical in corn seeds. A sample of crushed corn seeds is taken and was measured twice. One measurement was done with the first instrument and the other measurement was done with the second. This whole process was repeated nine times. The results in parts per billion are recorded:

Sample #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Instrument 1	3	8	9	4	6	7	5	6	8
Instrument 2	4	7	6	3	5	4	8	5	4
$d_i = I1 - I2$	-1	1	3	1	1	3	-3	1	4

1 $\bar{d} = 1.1111$, $S_d = 2.14735$ 1

a. Construct a 90% interval estimate for the mean difference in instrument readings.

90% C.I. $\Rightarrow t_{\frac{\alpha}{2}, n-1} = t_{0.025, 8} = 1.86$ 1 $\Rightarrow$

A 90% C.I. for $\mu_D = \left[\bar{d} \pm t_{0.025, 8} \frac{S_d}{\sqrt{n}}\right]$ 1

$= 1.1111 \pm (1.86) \frac{2.14735}{\sqrt{9}}$ 1

$= 1.1111 \pm 1.3314$

$= [-0.2203, 2.4425]$ 1

b. What assumption(s) did you **need** to construct the interval in (a)?

We need to assume that

1 The two samples are Dependent.

1 The differences population is Normal.

With the Best Wishes