KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
DHAHRAN, SAUDI ARABIA

STAT 319: Probability & Statistics for Engineers & Scientists

Semester 141

Second Major Exam

Wednesday November 5, 2014

5:30 – 7:00 pm

Please circle your instructor name:

Abbas	Al-Sabah	Al-Sawi
Anabosi	Malik	Saleh

Name: KEY ID #: 0000 Section #: Serial #:

Question No	Full Marks	Marks Obtained
1	5	
2	6	
3	20	
4	8	
5	11	
Total	50	

NOTE: Do NOT detach the formulae sheet; you can use the back of it to complete your solution for any question.

Q1. (2+3 marks) Consider the following function $f(x) = \begin{cases} 2(1-x), & 0 < x < 1 \\ 0 \quad , & otherwise \end{cases}$

a. Verify that it is a probability density function of some random variable X.

$$\int_0^1 f(x)\,dx = \int_0^1 2(1-x)\,dx \quad 1$$

$$= 2x - x^2 \Big]_0^1 = 2 - 1 = \boxed{1} \quad 1$$

So, $f(x)$ is a p.d.f. 1

b. Compute the average value of X.

$$E(X) = \mu_x \quad 1 \quad \int_0^1 x f(x)\,dx = \int_0^1 2x(1-x)\,dx$$

$$= x^2 - \frac{2}{3}x^3 \Big]_0^1 = 1 - \frac{2}{3} = \boxed{\frac{1}{3}} \quad 1$$

Q2. (6 marks)The number of viewers ordering a particular pay-per-view program is normally distributed. Twenty percent of the time, fewer than 20,000 people order the program. Only ten percent of the time more than 28,000 people order the program. What is the mean and standard deviation of the number of people ordering the program?

Let X: # of viewers $\Rightarrow X : N(\mu, \sigma)$

1 $P(X < 20{,}000) = 0.2 = P\left(Z < \frac{20{,}000 - \mu}{\sigma}\right)$ 1 $\Rightarrow$

$\frac{20{,}000 - \mu}{\sigma} = -0.84$ or $\mu - 0.84\sigma = 20{,}000$ --- (1)

1 $P(X > 28{,}000) = 0.1 = P\left(Z > \frac{28{,}000 - \mu}{\sigma}\right) \Rightarrow$ 1

$\frac{28{,}000 - \mu}{\sigma} = 1.28$ or $\mu + 1.28\sigma = 28{,}000$ --- (2)

Solving (1) & (2) simultaneously gives:

1 $\mu = 23{,}169.8113$, $\sigma = 3{,}773.5849$ 1

Q3. (12+3+5 marks) The following data represent the weights in pounds of a sample of 25 workers:

107	121	134	145	146	148	151	152	154
156	156	157	162	163	164	165	168	168
169	171	172	173	174	174	177		

where $\sum_{i=1}^{25} x_i = 3927$, $\sum_{i=1}^{25} x_i^2 = 623811$. Answer the following questions:

a. Construct a boxplot for the weights identifying outliers (if any) and comment on the shape of the boxplot.

a0) $\bar{x} = \frac{\sum x}{n} = \frac{3927}{25} = \boxed{157.08}$ pounds.

$s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}} = \sqrt{\frac{623811 - \frac{(3927)^2}{25}}{24}} = \boxed{17.0267}$ pounds

a1)

Stem	Leaf
10	7
11	
12	1
13	4
14	5 6 8
15	1 2 4 6 6 7
16	2 3 4 5 8 8 9
17	1 2 3 4 4 7

it has 3 modes (156, 168, 174) and -vely skewed

a2) $R_{78} = \frac{78}{100} \times 26 = 20.28$

$P_{78} = x_{(20)} + 0.28(x_{(21)} - x_{(20)})$

$= 171 + 0.28(172 - 171)$

$= \boxed{171.78}$ pounds

78% of the workers weigh less than or equal to 171.28 pt.

b. Construct a frequency distribution for the data, using five class intervals, and (135, 150] is one of the classes.

i	classes	f	x	xf	x^2f
1	(105, 120]	1	112.5	112.5	12656.25
2	(120, 135]	2	127.5	255	32512.5
3	(135, 150]	3	142.5	427.5	60918.75
4	(150, 165]	10	157.5	1575	248062.5
5	(165, 180]	9	172.5	1552.5	267806.25
	Total	25		3922.5	621956.25

c. Approximate the mean and the variance of the weights from the frequency distribution constructed in (b) before.

$\bar{x}_w = \frac{\sum xf}{\sum f} = \frac{3922.5}{25} = \boxed{156.9}$ pounds

$s_w^2 = \frac{\sum x^2 f - \frac{(\sum xf)^2}{n}}{n-1} = \frac{621956.25 - \frac{(3922.5)^2}{25}}{24}$

$= \frac{6516}{24} = \boxed{271.5}$ pounds

Q4. (3+3+2 marks) On average, there are 3.2 defects per 100 square meters of some fabric. Assume that the number of defects follows a Poisson distribution.

a. What is the probability that the next defect will be seen in less than 30 square meters?

Let X: Area until the next defect. $X: Exp(\frac{3.2}{100})$ (1)

$P(X<30) = 1 - e^{-\frac{3.2}{100}\times 30} = \boxed{0.6171}$ (1) (1)

b. Given that it takes more than 40 square meters to see the next defect, what is the probability that the next defect will be seen in less than 80 square meters?

$P(X<80 \mid X>40) = \frac{P(40<X<80)}{P(X>40)} = \frac{e^{-\frac{3.2\,40}{100}} - e^{-\frac{3.2\,80}{100}}}{e^{-\frac{3.2\,40}{100}}}$ (1) (1)

$= \frac{0.278 - 0.0773}{0.278} = \boxed{0.7219}$ (1)

or by lack of memory property

$P(X<40) = 1 - e^{-\frac{3.2\,40}{100}}$

$= 0.7219$

c. What is the expected area to see the next defect?

$E(X) = \mu_x = \frac{1}{\lambda} = \frac{100}{3.2} = \boxed{31.25}\, m^2$ (1) (1)

Q5. (4+3+4 marks) The fraction strength of a certain type of glass has an average of 14 (in thousands of pounds per square inch) and have a standard deviation of 2. If a sample of 100 pieces of this glass is randomly selected then,

a. What is the probability that the average fraction strength for the 100 pieces exceeds 14.5?

X: Fraction strength. $\mu_x = 14$, $\sigma_x = 2$, $n = 100$

(1) $P(\bar{X} > 14.5) = P(Z > \frac{14.5 - 14}{0.2}) = P(Z > 2.5)$ (1) (1)

(1) $= \boxed{0.0062}$ $= P(Z < -2.5)$

b. Given that the fraction strength is normally distributed, what is the probability that a randomly selected piece will have a strength more than 13?

$X: N(14, 2)$,

(1) $P(X > 13) = P(Z > \frac{13-14}{2}) = P(Z > -0.5)$ (1)

$= P(Z < +0.5) = \boxed{0.6915}$ (1)

c. What is the approximate probability that at most 45 pieces, out of the 100, each will have a strength of no more than 13 thousand pounds per square inch?

Y: # of pieces with strength ≤ 45. $Y \sim B(100, 0.3085)$

$\mu = np = 100(0.3085) = 30.85$ (1)

$\sigma = \sqrt{npq} = \sqrt{100(0.6915)(0.3085)} = \sqrt{21.33} = 4.6187$ (1)

$P(Y_B \leq 45) \approx P(Y_N \leq 45.5) = P(Z \leq \frac{45.5 - 30.85}{4.6187})$ (1) (1)

$\approx P(Z < 3.17) = \boxed{0.9992}$