KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES
DHAHRAN, SAUDI ARABIA

STAT 319: Probability & Statistics for Engineers & Scientists
Semester 141
First Major Exam
Wednesday October 15, 2014
6:00 – 7:00 pm

Please circle your instructor name:

Abbas	Al-Sabah	Al-Sawi
Anabosi	Malik	Saleh

Name: KEY ID #: 0000 Section #: Serial #:

Question No	Full Marks	Marks Obtained
1	6	
2	6	
3	14	
4	6	
5	13	
Total	45	

Q1. (6 marks) A manufacturing firm employs three analytical plans for the design and development of a particular product. For cost reasons, all three are used at different times. In fact, plans 1 and 2 are used for 30% and 20% of the products, respectively. The defect rate for the plans 1, 2, and 3 are 1%, 3%, and 2% respectively. If a random product was observed and found to be defective, which plan was most likely used and thus responsible?

(2) $\begin{cases} P(I) = 0.3, & P(II) = 0.2, & P(III) = 0.5 \\ P(D|I) = 0.01, & P(D|II) = 0.03, & P(D|III) = 0.02 \end{cases}$

(3) $\begin{cases} P(D \cap I) = P(D|I) \cdot P(I) = 0.3 \times 0.01 = 0.003 \\ P(D \cap II) = P(D|II) \cdot P(II) = 0.2 \times 0.03 = 0.006 \\ P(D \cap III) = P(D|III) \cdot P(III) = 0.5 \times 0.02 = \boxed{0.01} \end{cases}$

(1) so, it is plan 3

Q2. (2+2+2 marks) Is each statement below True or False? Give an explanation.

F a. The probability that a mineral sample will contain silver is 0.38 and the probability that it will not contain silver is 0.52.

$P(A^c) = 1 - P(A) \Rightarrow$ if $P(A) = 0.38 \Rightarrow$

$P(A^c) = 0.62 \neq 0.52$

F b. The probability that a student will get an A in STAT 319 is 0.3, and the probability that he will get either and A or a B is 0.27.

$P(A \cup B) \geq P(A)$ for <u>any</u> two events A & B

F c. A company is constructing two buildings; the probability that the larger one will be completed on time is 0.35 and the probability that both will be completed on time is 0.42.

Since $A \cap B \subseteq A$ & $A \cap B \subseteq B \Rightarrow$

$P(A \cap B) \leq P(A)$ & $P(A \cap B) \leq P(B)$

Q3. (2+2+2+8 marks) Errors in an experimental transmission channel are found when the transmission is checked by a certifier that detects missing pulses. The number of errors found in an eight-bit byte is a random variable with the following cumulative distribution function:

$$F(x) = \begin{cases} 0, & x < 1 \\ 0.8, & 1 \le x < 4 \\ 0.9, & 4 \le x < 7 \\ 1.0, & 7 \le x \end{cases}$$

Determine each of the following probabilities.

2 a. $P(X \le 4) =$ $F(4) = \boxed{0.9}$

2 b. $P(X > 7) =$ $1 - P(X \le 7) = 1 - F(7) = 1 - 1 = \boxed{0}$

2 c. $P(X \le 5) =$ $F(5) = F(4) = \boxed{0.9}$

d. Find the mean and variance of the number of errors found in an eight-bit byte.

2 $f(1) = 0.8 - 0 = 0.8$, $f(4) = 0.9 - 0.8 = 0.1$, $f(7) = 1 - 0.9 = 0.1$

x	$f(x)$	$x f(x)$ (2)	$x^2 f(x)$ (2)
1	0.8	0.8	0.8
4	0.1	0.4	1.6
7	0.1	0.7	4.9
Total	1	1.9	7.3

$\Rightarrow E(X) = \sum_{\forall x} x f(x) = \boxed{1.9}$ 1

$E(X^2) = \sum_{\forall x} x^2 f(x) = 7.3$

$V(X) = E(X^2) - E^2(X)$

$= 7.3 - 1.9^2 = \boxed{3.69}$ 1

Q4. (4+2 marks) Assume that flaws per sheet of glass can be represented by a Poisson distribution, with an average of 0.7 flaws per sheet.

a. What is the probability that randomly selected two sheets of glass have more than one flaw?

Let X: # of flaws per sheet. $X: Po(\lambda t)$, where $\lambda = 0.7$

If $t = 2$ (1) $\Rightarrow f(x) = \dfrac{e^{-1.4}\, 1.4^{x}}{x!}$, $x = 0, 1, 2, \ldots$ 1

$P(X > 1) = 1 - P(X \le 1) = 1 - [f(0) + f(1)]$ 1

$= 1 - e^{-1.4}[1 + 1.4] = 1 - 2.4e^{-1.4} = \boxed{0.4082}$ 1

b. What is the mean number of flaws per 12 sheets?

$\mu_X = E(X) = \lambda t = (0.7)\,12 = \boxed{8.4}$ flaws

1 1

Q5. (4+4+3+2 marks) Twenty five percent of all households have a DVD player.

a. If you select 20 houses at random, what is the probability that at least three of them have a DVD player?

(1)

$p = 0.25$, $n = 20$, X: # of houses with DVD $\Rightarrow X : B(20, 0.25)$

(1) $f(x) = {}_{20}C_x \, 0.25^x \, 0.75^{20-x}$, $x = 0, 1, 2, \ldots, 20$

$P(X \geq 3) = 1 - P(X < 3) = 1 - P(X \leq 2)$

$= 1 - [f(0) + f(1) + f(2)]$ (1)

$= 1 - [0.75^{20} + {}_{20}C_1 \, 0.25 \, 0.75^{19} + {}_{20}C_2 \, 0.25^2 \, 0.75^{18}]$

$= 1 - [0.0032 + 0.0211 + 0.0669] = \boxed{0.9088}$ (1)

b. If the household were checked one by one, what is the probability that the first household, that has a DVD player, is the fifth?

(1) Let X: # of houses checked untill the 1^{st} with a DVD.

$X : G(0.25) \Rightarrow$ (1) $f(x) = (0.25)(0.75)^{x-1}$, $x = 1, 2, \ldots$

$P(X=5) = f(5) = (0.25)(0.75)^{5-1} = \boxed{0.0791}$ (1) (1)

c. Given that in a randomly selected block there are 16 houses, what is the probability that 3 houses would have a DVD player, in a sample of 8 houses randomly selected from that block?

$N = 16$, $n = 8$, $x = 3$, $K = 0.25 \times 16 = 4$.

(1) Let X: # of houses with DVD in a sample of 8 out of 16.

$\Rightarrow f(x) = \dfrac{{}_4C_x \cdot {}_{12}C_{8-x}}{{}_{16}C_8} \Rightarrow$

(1)

$P(X=3) = {}_4C_3 \cdot {}_{12}C_5 / {}_{16}C_8 = \dfrac{4 \times 792}{12870} = \boxed{0.2462}$ (1)

d. Referring to part c before, what is the expected number of houses that would have a DVD player?

(1) (1)

$E(X) = \mu_X = \dfrac{n}{N} K = \dfrac{8}{16} \times 4 = \boxed{2}$ households

With the Best Wishes