Formula Sheet

$$\bar{x} \equiv \frac{1}{n}\sum x; \qquad s_{xx} \equiv \sum (x-\bar{x})^2 = \sum x^2 - \frac{1}{n}\left(\sum x\right)^2; \qquad s^2 \equiv \frac{s_{xx}}{n-1}.$$

$P(A \cup B) = P(AB') + P(A'B) + P(AB).$

$P(A \cup B) = P(A) + P(B) - P(AB).$

$P(A \cup B) = 1 - P(A \cup B)' = \; 1 - P(A'B').$ De Morgan's Law

$P(AB)' = P(A' \cup B').$ De Morgan's Law

$$P(A \mid B) = \frac{P(AB)}{P(B)}, \quad P(B) \neq 0; \quad P(AB) = P(A)P(B \mid A) = P(B)P(A \mid B)$$

Independence: $P(AB) = P(A)P(B).$.

$$\mu \equiv E(X) = \sum_x x\, f(x).$$

$$E(X^2) = \sum x^2\, f(x), \;\; \sigma^2 \equiv E(X-\mu)^2 = E(X^2) - \mu^2.$$

The Binomial Distribution: $f(x) = \binom{n}{x} p^x (1-p)^{n-x}, \;\; x = 0,1,\ldots,n; \; 0 < p < 1; \;\; \mu = np, \; \sigma^2 = npq.$

The Geometric Distribution: $f(x) = q^x p, \;\; x = 0,1,2,\cdots; \; q = 1-p; \;\; \mu = 1/p, \; \sigma^2 = q/p^2.$

The Hypergeometric Distribution

$$f(x) = \binom{K}{x}\binom{N-K}{n-x} \div \binom{N}{n}, \;\; \mu = np, \;\; \sigma^2 = (1-c)\, npq, \;\; (N-1)\, c = n-1, \; p = (K/N), \;\; q = 1-p.$$

The Poisson Distribution: $f(x) = \dfrac{(\lambda t)^x}{x!} e^{-\lambda t}; \;\; x = \; 0,1,\ldots; \; \mu = \lambda t, \; \sigma^2 = \lambda t.$

$P(a < X < b) = \int_a^b f(x)dx; \quad P(X \leq k) = \int_{-\infty}^{k} f(x)dx$ where k is a particular value of x.

$$\mu \equiv E(X) = \int_{-\infty}^{\infty} x\, f(x)dx., \qquad E(X^2) = \int_{-\infty}^{\infty} x^2\, f(x)dx, \quad \sigma^2 \equiv V(X) = E(X^2) - \mu^2.$$

The Normal Distribution $X \sim N(\mu, \sigma^2), \;\; Z = \dfrac{X-\mu}{\sigma} \sim N(0,1)$

The Exponential Distribution: $f(x) = \dfrac{1}{\beta}\, e^{-x/\beta}, \;\; 0 \leq x; \; \mu = \beta, \;\; \sigma^2 = \beta^2.$

Confidence Interval	**Test Statistic**
$\bar{x} \mp z_{\alpha/2}\left(\dfrac{\sigma}{\sqrt{n}}\right).,\qquad n = \dfrac{z_{\alpha/2}^2}{e^2}\sigma^2,$	$z = \dfrac{\bar{x}-\mu_0}{\dfrac{\sigma}{\sqrt{n}}}$
$\bar{x} \mp t_{\alpha/2}\left(\dfrac{s}{\sqrt{n}}\right)$	$t = \dfrac{\bar{x}-\mu_0}{\sqrt{s^2/n}}\ ,\quad df = n-1.$
$(\bar{x}_1-\bar{x}_2) \mp z_{\alpha/2}\sqrt{\dfrac{\sigma_1^2}{n_1}+\dfrac{\sigma_2^2}{n_2}}.$	$z = \dfrac{\bar{x}_1-\bar{x}_2-\delta_0}{\sqrt{\left(\sigma_1^2/n_1\right)+\left(\sigma_2^2/n_2\right)}}.$
$(\bar{x}_1-\bar{x}_2) \pm t_{\alpha/2}\cdot s_p\cdot\sqrt{\frac{1}{n_1}+\frac{1}{n_2}},\qquad df = n_1+n_2-2$	$t = \dfrac{(\bar{x}_1-\bar{x}_2)-\delta_0}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}\ ,\qquad s_p^2 = \dfrac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$
$(\bar{x}_1-\bar{x}_2) \pm t_{\alpha/2}\cdot\sqrt{\dfrac{s_1^2}{n_1}+\dfrac{s_2^2}{n_2}}$	$t = \dfrac{(\bar{x}_1-\bar{x}_2)-\delta_0}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_1^2}{n_2}}}\qquad \dfrac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{\left(s_1^2/n_1\right)^2}{n_1-1}+\frac{\left(s_2^2/n_2\right)^2}{n_2-1}}$
$\bar{d} \pm t\dfrac{s_d}{\sqrt{n}}$	$t = \dfrac{\bar{d}}{s_d/\sqrt{n}}$
$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\qquad n = \dfrac{z_{\alpha/2}^2}{e^2}\hat{p}\hat{q}$	$z = \dfrac{\hat{p}-p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$
$(\hat{p}_1-\hat{p}_2) \pm z\sqrt{\dfrac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\dfrac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$	$z = \dfrac{\hat{p}_1-\hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}},\qquad \hat{p} = \dfrac{x_1+x_2}{n_1+n_2}$

$$y=\beta_0+\beta_1 x+\varepsilon\,,\qquad \hat{y}.=\hat{\mu}(x)=\hat{\beta}_0+\hat{\beta}_1 x\,,\qquad \hat{y}.=\hat{\mu}(x)\,,\qquad e=y-\hat{\mu}(x).$$

$$s_{yy}=\sum y^2-\frac{1}{n}\left(\sum y\right)^2,\qquad s_{xy}=\sum xy-\frac{1}{n}\left(\sum x\right)\left(\sum y\right),\ \ s_{xx}=\sum x^2-\frac{1}{n}\left(\sum x\right)^2$$

$$\hat{\beta}_1=b_1=\frac{s_{xy}}{s_{xx}}\qquad\qquad \hat{\beta}_0=b_0=\bar{y}-\hat{\beta}_1\bar{x}\,.$$

$$SSR=\hat{\beta}_1 s_{xy},\qquad SSE=s_{yy}-SSR.=SST-SSR.$$

$$r=\frac{s_{xy}}{\sqrt{s_{xx}\ s_{yy}}}.,\qquad \left(r\sqrt{s_{yy}}=\hat{\beta}_1\sqrt{s_{xx}}\right)$$

$$MSE=s^2=\frac{SSE}{n-2}$$

$$R^2=\frac{\text{Explained variation}}{\text{Total variation}}=\frac{SSR}{s_{yy}}.$$

$\hat{\beta}_0 \mp t_{\alpha/2}\sqrt{\left(\dfrac{1}{n}+\dfrac{\bar{x}^2}{s_{xx}}\right)MSE}.,\qquad df=n-2$	$H_0:\beta_0=c\,,\qquad t=\dfrac{b_0-c}{\sqrt{s^2\left(\frac{1}{n}+\frac{\bar{x}^2}{s_{xx}}\right)}}$
$\hat{\beta}_1 \mp t_{\alpha/2}\sqrt{\dfrac{MSE}{s_{xx}}}.$	$H_0:\beta_1=c\,,\qquad t=\dfrac{\hat{\beta}_1-c}{\sqrt{MSE/s_{xx}}}.$
	$H_0:\rho=0\,,\qquad t=\dfrac{r\sqrt{n-2}}{\sqrt{1-r^2}}.\qquad \left(\rho\sigma_y=\beta_1\sigma_x\right)$
$\hat{\mu}(x)\ \pm\ t_{\alpha/2}\sqrt{\left(\dfrac{1}{n}+\dfrac{(x-\bar{x})^2}{s_{xx}}\right)MSE}.$	$\hat{\mu}(x)\pm\ t_{\alpha/2}\sqrt{\left(1+\dfrac{1}{n}+\dfrac{(x-\bar{x})^2}{s_{xx}}\right)MSE}.$