Department of Mathematics and Statistics
Semester 141

STAT302 Final Exam Tuesday December 30, 2014

Name: _____ ID #: _____

**Instructions:**

- Justify your work
- State theorems and results you are using
- Show all details
- In hypothesis testing problems, write down the hypotheses, the rejection region, the decision and the conclusion.
- You will lose points for violations of the above rules

| Question | Marks | Marks Obtained |
|---|---|---|
| 1 | 5 | |
| 2 | 7 | |
| 3 | 6 | |
| 4 | 4 | |
| 5 | 6 | |
| 6 | 5 | |
| 7 | 12 | |
| Total | 45 | |

1) Let $Y_1, \cdots, Y_n$ be a random sample from a uniform distribution on $(\theta, \theta + 1)$.

a) Is $\bar{Y}$ an unbiased estimator of $\theta$? If yes, prove it. If no, find the bias. *(2pts.)*

b) If $\bar{Y}$ is biased, find a function of $\bar{Y}$ which is unbiased. *(1pt.)*

c) Find $MSE(\bar{Y})$. *(2pts.)*

2) Let $Y$ be a single observation from the distribution

$$f_Y(y|\theta) = \begin{cases} \theta y^{\theta-1}, & 0 < y < 1 \\ 0 & otherwise \end{cases} \quad ; \quad \theta > 0$$

a) Show that $y^\theta$ is a pivotal quantity. *(3pts.)*

b) Use this pivotal quantity to find a symmetric 90% confidence interval for $\theta$. *(4pts.)*

3) Let $Y_1, \cdots, Y_n$ be a random sample from a population with density

$$f_Y(y|\theta) = \begin{cases} \theta y^{\theta-1}, & 0 < y < 1 \\ 0 & otherwise \end{cases} \qquad \theta > 0$$

a) Show that $\bar{Y}$ is an unbiased estimator of $\frac{\theta}{\theta+1}$. *(2pts.)*

b) Is $\bar{Y}$ a consistent estimator of $\frac{\theta}{\theta+1}$.? Explain. *(4pts.)*

4) For testing $H_0: \theta = \theta_0 \ \ vs\ H_a: \theta > \theta_0$,

a) Give the rejection region for a large sample level α test. *(1pt.)*

b) Give a large sample $100(1-\alpha)\%$ lower confidence bound for $\theta$. *(1pt.)*

c) What is the relationship between the two procedures? Explain. *(2pts.)*

5) Let $Y_1, \cdots, Y_n$ be a random sample from a population with density

$$f_Y(y|\theta) = \begin{cases} \frac{3y^2}{\theta^3}, & 0 \le y \le \theta \\ 0 & otherwise \end{cases} \quad ; \quad \theta > 0$$

a) Find a method of moments estimator for $\theta$. *(2pts.)*

b) Find a sufficient statistic for $\theta$. *(2pts.)*

c) Is the method of moments estimator found in a) above MVUE? Explain. *(2pts.)*

6) Let $Y_1, \cdots, Y_n$ be a random sample from a population with density *(5pts.)*

$$f_Y(y|\theta) = \begin{cases} \frac{1}{\theta} e^{-y/\theta}, & y > 0 \\ 0 & otherwise \end{cases} \quad ; \quad \theta > 0$$

Derive the most powerful test for $H_0: \theta = \theta_0$ $vs$ $H_a: \theta = \theta_a$, where $\theta_a < \theta_0$.

7) An insurance company conducted a study on the relationship between lung disease and air pollution. Random samples of 400 adults from each of 4 cities gave the following results:

| City | Number of adults with lung disease |
|---|---|
| Old Jubail | 34 |
| Jubail Industrial City | 42 |
| Dammam | 21 |
| Khobar | 18 |

At the 5% significance level, do the data provide sufficient evidence to indicate a difference in the proportions with lung disease for the 4 cities? *(12pts.)*