Department of Mathematics and Statistics
Semester 141

STAT302 Major Exam #3 Wednesday December 17, 2014

Name: _____ ID #: _____

- Justify your work – state theorems and results you are using
- Show all details

1) True or False? *(3 pts.)*
   a) The level of the test is the probability that the null hypothesis is true.
   b) The probability of type II error is the probability that the alternative hypothesis is true.
   c) In a credibility interval the end points are random.

2) A group of 35 AS students took a statistics test and the average was 32 with a standard deviation of 4.3. An independent sample of management students took the same test and their average was 31.5 and a standard deviation of 4.5.
   a) At the 5% significance level, test whether the AS students have a higher mean test score. *(4 pts.)*

   b) Find the probability of type II error when the difference between the true means is 3. *(3 pts.)*

   c) For the same test above, what is the sample size needed, assuming equal-size samples for each group, to attain a power of 0.95 to distinguish between means with a difference of 3 units. *(2 pts.)*

3) If $Y_1, Y_2, \dots, Y_n$ is a random sample from $f(y|\theta) = \begin{cases} \theta e^{-\theta y} & y > 0 \\ 0 & otherwise \end{cases}$. $\theta > 0$

Find the level α likelihood ration test for $H_0 : \theta \leq \theta_0 \; vs. H_a : \theta > \theta_0$. *(7 pts.)*

4) If $Y_1, Y_2, \dots, Y_n$ is a random sample from *N(θ,1)*, and suppose that θ has a uniform $U(-1, 1)$ prior.

a) Find the posterior distribution of $\theta|Y_1, Y_2, \dots, Y_n$. *(5 pts.)*

b) Can you identify the posterior? If yes, what is it? *(2 pts.)*

c) Is $U(-1, 1)$ a conjugate prior? Explain. *(1 pts.)*

d) Find the Bayes' estimator of θ^2. *(3 pts.)*