Department of Mathematics and Statistics
Semester 141

STAT302 Second Major Exam Monday November 17, 2014

Name: ______________________________ ID #: ____________________

1) If $\hat{\theta}$ is a biased estimator for θ with bias 5, what is $E(\hat{\theta})$?

2) Let Y have the probability density function

$$f_Y(y|\theta) = \begin{cases} \dfrac{2(\theta - y)}{\theta^2}, & 0 < y < 1 \\ 0 & otherwise \end{cases}$$

Find a 90% upper confidence limit for θ.

3) $Y_1, \cdots, Y_n$ is a random sample from a distribution with variance σ^2, and let

$S_1^2 = \frac{1}{n} \sum_1^n (Y_i - \bar{Y})^2$ and $S_2^2 = \frac{1}{(n-1)} \sum_1^n (Y_i - \bar{Y})^2$.

S_1^2 is superior to S_2^2 in regard to variance, while S_2^2 is superior to S_1^2 in regard to bias.

Considering the two criteria, bias and variance, together, which is the better estimator?

Hint: Think of a measure that incorporates both criteria.

4) $Y_1, \cdots, Y_n$ is a random sample from a Poisson distribution with parameter λ. Consider

$\hat{\lambda}_1 = \frac{Y_1 + Y_2}{2}$, and $\hat{\lambda}_2 = \bar{Y}$. Find the efficiency of $\hat{\lambda}_1$ relative to $\hat{\lambda}_2$.

5) $Y_1, \cdots, Y_n$ are iid random variables from the following density

$$f_Y(y|\alpha,\theta) = \begin{cases} \dfrac{\alpha y^{\alpha-1}}{\theta^{\alpha}}, & 0 \le y \le \theta \\ 0, & otherwise \end{cases}$$

If θ is known, find a sufficient statistic for α.

6) $Y_1, \cdots, Y_n$ are iid random variables from the following density

$$f_Y(y|\theta) = \begin{cases} \dfrac{1}{2\theta+1}, & 0 \le y \le 2\theta + 1 \\ 0, & otherwise \end{cases}$$

Find the MLE for the variance of Y.

7) $Y_1, \cdots, Y_n$ are iid random variables from the following density

$$f_Y(y|\theta) = \begin{cases} \dfrac{2y}{\theta} e^{-\frac{y^2}{\theta}}, & y > 0 \\ 0, & otherwise \end{cases}$$

Find MVUE for θ.