Department of Mathematics and Statistics
Semester 141

STAT302 First Major Exam Sunday September 21, 2014

Name: _____ ID #: _____

- Define your variables
- Justify your work
- Give reasons
- Show Details

1) Suppose the life in hours, *X*, of a light bulb manufactured by company *A* is *N(800, 14,400)*. Independently, the life in hours, *Y*, of a light bulb manufactured by company *B* is *N(850, 2500)*. One bulb is selected at random from each company and used until failure; find the probability that the life of the light bulb from company *A* exceeds the life of the light bulb from company *B* by at least 15 hours.

2) $Y_{(1)}, \ldots, Y_{(5)}$ are the order statistics from a sample of size $n = 5$, from the following distribution $f_Y(y) = \begin{cases} 2y, & 0 < y < 1 \\ 0 & otherwise \end{cases}$

a) Find $P\left(Y_{(4)} < \frac{1}{2}\right)$ without using the pdf or cdf of $Y_{(4)}$.

b) Generalize the result in (a) to find the cdf of $Y_{(4)}$.

c) Use the result in (b) to show that the pdf of $Y_{(4)}$ is $\frac{5!}{3!1!}[F_Y(y)]^3[1 - F_Y(y)]f_Y(y)$

3) Find the expected value and variance of a *t* random variable.

Hint: Use the fact that if *Y* has a χ^2 distribution with ν degrees of freedom then

$E(Y^a) = \frac{\Gamma\left(\frac{\nu}{2}+a\right)}{\Gamma\left(\frac{\nu}{2}\right)}, if\ \nu > 2$, and the properties of the gamma function.