HW4
Convex Analysis (Math 580)

Term 141, Year 2014/15

Problem 1:

Let Ω_1 and Ω_2 be nonempty, convex subsets of $\mathbb{R}^n$. Suppose that $\operatorname{ri}\Omega_1 \cap \operatorname{ri}\Omega_2 \neq \emptyset$. Prove that

$$N(\bar{x};\Omega_1 \cap \Omega_2) = N(\bar{x};\Omega_1) + N(\bar{x};\Omega_2) \text{ for all } \bar{x} \in \Omega_1 \cap \Omega_2.$$

Problem 2:

Let $f_1, f_2\colon \mathbb{R}^n \to \overline{\mathbb{R}}$ be convex functions satisfying the condition

$$\operatorname{ri}(\operatorname{dom} f_1) \cap \operatorname{ri}(\operatorname{dom} f_2) \neq \emptyset.$$

Prove the for all $\bar{x} \in \operatorname{dom} f_1 \cap \operatorname{dom} f_2$ we have

$$\partial(f_1 + f_2)(\bar{x}) = \partial f_1(\bar{x}) + \partial f_2(\bar{x}), \quad \partial^\infty(f_1 + f_2)(\bar{x}) = \partial^\infty f_1(\bar{x}) + \partial^\infty f_2(\bar{x}).$$

Problem 3:

Find the Fenchel conjugate for each of the following functions on $\mathbb{R}$:

(i) $f(x) = e^x$.

(ii) $f(x) = \begin{cases} -\ln(x) & x > 0, \\ \infty & x \leq 0. \end{cases}$

Problem 4:

Define the function $f\colon \mathbb{R} \to \overline{\mathbb{R}}$ by

$$f(x) := \begin{cases} -\sqrt{1 - x^2} & \text{if } |x| \leq 1, \\ \infty & \text{otherwise.} \end{cases}$$

Show that f is a convex function and calculate its directional derivatives $f'(-1;d)$ and $f'(1;d)$ in the direction $d = 1$.