HW3
Convex Analysis (Math 580)

Term 141, Year 2014/15

Problem 1:

i. Let Ω_1 and Ω_2 be nonempty, convex subsets of $\Re^n$ and $\Re^p$, respectively. For $(\bar{x}_1, \bar{x}_2) \in \Omega_1 \times \Omega_2$, show that

$$N((\bar{x}_1, \bar{x}_2); \Omega_1 \times \Omega_2) = N(\bar{x}_1; \Omega_1) \times N(\bar{x}_2; \Omega_2).$$

ii. Let Ω_1 and Ω_2 be convex subsets of $\Re^n$ with $\bar{x}_i \in \Omega_i$ for $i = 1, 2$. Show that

$$N(\bar{x}_1 + \bar{x}_2; \Omega_1 + \Omega_2) = N(\bar{x}_1; \Omega_1) \cap N(\bar{x}_2; \Omega_2).$$

Problem 2:

Let $\Omega := \left\{ x \in \Re^n \mid x \leq 0 \right\}$. Show that

$$N(\bar{x}; \Omega) = \left\{ v \in \Re^n \mid v \geq 0, \langle v, \bar{x} \rangle = 0 \right\} \text{ for any } \bar{x} \in \Omega.$$

Problem 3:

Let $f : \Re^n \to \Re$ be a convex function differentiable at $\bar{x}$. Show that

$$N((\bar{x}, f(\bar{x})); \operatorname{epi}(f)) = \left\{ \lambda \left(\nabla f(\bar{x}), -1 \right) \mid \lambda \geq 0 \right\}.$$

Problem 4:

Calculate the subdifferentials and the singular subdifferentials of the following convex function at every point of their domains:

(a) $f(x) = |x-1| + |x-2|, \qquad x \in \Re.$

(b) $f(x) = e^{|x|}, \qquad x \in \Re.$