HW2
Convex Analysis (Math 580)

Term 141, Year 2014/15

Problem 1:

Show that the set $\{(x,t) \in \Re^n \times \Re \mid t \geq \|x\|\}$ is closed and convex.

Problem 2:

The *indicator function* associated with a set $C \in \Re^n$ is defined by

$$\delta(x;C) := \begin{cases} 0, & \text{if } x \in C, \\ \infty, & \text{otherwise.} \end{cases}$$

(i) Calculate $\delta(\cdot;C)$ for $C = \varnothing$, $C = \Re^n$, and $C = [-1,1]$.

(ii) Show that the set C is convex set if and only if its indicator function $\delta(\cdot;C)$ is convex.

Problem 3:

Let X be a nonempty bounded subset of $\Re^n$. Show that $\mathrm{cl}(\mathrm{conv}(X)) = \mathrm{conv}(\mathrm{cl}(X))$. In particular, if X is compact, then $\mathrm{conv}(X)$ is compact.

Problem 4:

Construct an example of a point in a nonconvex set X that has the prolongation property, but is not a relative interior point of X .

Problem 5:

Show that a function $f : \Re^n \to [-\infty, \infty]$ is lower semicontinuous at $x \in \Re^n$ if and only if for every $M \in \Re$ with $f(x) > M$ there exists $\delta > 0$ such that

$$f(y) > M \qquad \text{for all} \quad y \in B_\delta(x).$$