King Fahd University of Petroleum and Minerals
Department of Mathematic

Math 580: Convex Analysis
Major Exam 2
Fall 2014
Time Limit: 120 Minutes

Student ID :
Student Name :

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	30	
3	20	
4	30	
Total	100	

1. (20 points) Let C be a nonempty, convex subset of $\Re^n$ and let $\bar{x}$ be a boundary point of C. Show that the sets C and $\{\bar{x}\}$ form an extremal system.

2. (30 points) Let $C \subset \Re^n$ be a convex set with $\bar{x} \in \Re^n$.

   (a) Write a definition of the *Normal Cone* to C at $\bar{x}$, $N(\bar{x}; C)$.

   (b) Prove that the normal cone to C at $\bar{x}$ is the singleton $\{0\}$ whenever $\bar{x} \in \text{int}(C)$.

3. (20 points) Let $f : \Re^n \to \overline{\Re}$ be a convex function and $\bar{x} \in \text{dom} f$. Show only <u>one</u> of the following:

   (a) $\partial^\infty(\bar{x}) = N(\bar{x}; \text{dom} f)$.

   (b) $\partial(\bar{x}) = \{v \in \Re^n : (v, -1) \in N((\bar{x}, f(\bar{x})); \text{epi} f)\}$.

4. (30 points) Consider the following convex function

$$f(x) = \begin{cases} x^2 - 1, & \text{if } |x| \le 1, \\ \infty, & \text{otherwise on } \mathfrak{R}. \end{cases}$$

Find

(a) $\partial^\infty f(0), \partial^\infty f(-1)$.

(b) $\partial f(0), \partial f(-1)$.