Final Version

King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 513 Final Exam
The First Semester of 2014-2015 (141)

Time Allowed: 180 Minutes

Name: _____ ID#: _____

Section/Instructor: _____ Serial #: _____

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		25
2		25
3		25
4		25
5		25
6		20
7		25
Total		170

Q:1 (25 points) Use Laplace transform method to solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = xe^{-t}, \quad 0 < x < \infty, \quad t > 0$$

with initial conditions $u(x,0) = 1, u_t(x,0) = 0, \quad 0 < x < \infty$
and the boundary conditions

$$u(0,t) = cos(t),\ lim_{x\to\infty}|u(x,t)| \quad x^n, \quad n \ \ finite, t > 0$$

Q:2 (25 points) Solve the heat equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < \pi, \quad t > 0$$

subject to the following initial and **non-homogeneous** boundary conditions

$$u(0,t) = 2,\ u_x(\pi,t) = 0, t > 0,\ u(x,0) = 4,\ 0 < x < \pi$$

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Q:3 (25 points) Solve the Laplace equation by separation of variables

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, 0 < x < 1, 0 < y < \pi$$

$$u(0, y) = y, \qquad u_y(x, y)|_{y=\pi} = 0$$

$$u_x(x, y)|_{x=1} = 0, \qquad u_y(x, y)|_{y=0} = 0.$$

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Q:4 (25 points) Solve

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} = \frac{\partial^2 u}{\partial t^2}, \quad 0 < r < 1, t > 0$$

subject to following boundary conditions

$$u(r,0) = 1 - r^2, \quad \frac{\partial u}{\partial t}|_{t=0} = 0, \quad u(1,t) = 0.$$

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Q:5 (25 points) Find the steady-state temperature in the sphere of radious C by solving

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{cot\theta}{r^2}\frac{\partial u}{\partial \theta} = 0, 0 < r < C, 0 < \theta < \pi$$

$$u(C, \theta) = 1 - cos(2\theta), 0 < \theta < \pi.$$

(Hint $P_0(x) = 1, P_1(x) = x, P_2(x) = \frac{1}{2}(3x^2 - 1)$).

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Q:6a (10 points) Given that $\mathcal{L}^{-1}\{\frac{s}{(s^2+1)^2}\} = \frac{1}{2}t\ sin(t)$, find $\mathcal{L}^{-1}\{\frac{1}{(s^2+1)^2}\}$.

Q:6b (10 points) Show that $J_2(x) = J_0''(x) - \frac{J_0'(x)}{x}$

Q:7 (25 points) Use the matrix exponential to find the general solution of the following system of first-order linear ordinary differential equations

$x' = x + y + 2z + t$
$y' = -x + 3y + 4z + 1$
$z' = 2z + e^t.$

(Hint: Set of fundamental solutions is $S = \{e^{2t},\ te^{2t},\ t^2e^{2t}\}$)
(Do not evaluate the integral)