Final Version

King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 513 Major Exam II
The First Semester of 2014-2015 (141)
Time Allowed: 105 Minutes

Name: ______________________ ID#: ______________

Section/Instructor: ______________ Serial #: ______________

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		10
2		16
3		12
4		14
5		12
6		12
7		14
Total		90

Q:1 (10 points) Show that the fourier transform of $f(t) = \begin{cases} cos(at) & |t| < 1 \\ 0 & |t| > 1 \end{cases}$

is $F(w) = \frac{sin(w\ -\ a)}{w\ -\ a} + \frac{sin(w\ +\ a)}{w\ +\ a}$.

Q:2a (8 points) Use partial fractions to invert the Fourier transform:

$$F(w) = \frac{1}{(1 + w)(1 + 2iw)^2}.$$

Q:2b (8 points) Use the fact that $F[e^{-3t}H(t)] = \frac{1}{3 + iw}$ and Parseval's equality to show that

$$\int_{-\infty}^{\infty} \frac{dx}{9 + x^2} = \frac{\pi}{3}.$$

Q:3 (12 points) Evaluate by Fourier transform $e^{-t}H(t) * e^{t}H(-t)$.

Q:4 (14 points) Solve the Sturm- Liouville problem:

$$y'' + \lambda y = 0, \qquad y(0) + y'(0) = 0, \quad y(\pi) + y'(\pi) = 0.$$

Q:5 (12 points) The Sturm-Liouville problem $y'' + \lambda y = 0, \quad y'(0) = y'(L) = 0$ has the eigen function solutions $y_0(x) = 1$ and $y_n(x) = cos(\frac{n\pi x}{L})$. Find the eigenfunction expansion for $f(x) = x$ using these eigenfunctions.

Q:6 (12 points)Find the expansion with **five** nonvanishing coefficients in Legendre polynomials of the function: $f(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & -1 < x < 0 \end{cases}$

Q:7 (14 points) Let $A = \begin{pmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{pmatrix}$

(a) Find the eigenvalues of A

(b)Find an eigenvector corresponding to $\lambda = -4$, Formula Sheet

If $f(t)$ is a function with Fourier transform $F(w)$, then

(I) $F[f(t-\tau)] = e^{-iw\tau}F(w)$

(II) $F[f(kt)] = F(w/k)/|k|$, k is a scalar

(III) $F[F(t)] = 2\pi f(-w)$

(IV) $F[f^{(n)}(t)] = (iw)^n F(w)$

(V) $F[f(t)e^{iw_0 t}] = F(w - w_0)$