KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 421: EXAM II, SEMESTER (141), DECEMBER 11, 2014

Math421: Introduction to Topology

Name : ....

ID : ....

Exercise	**Points**
1	______ / 20
2	______ / 10
3	______ / 30
4	______ / 30
5	______ / 10
Total	______ / 100

Exercise 1. Let $(X, \mathcal{T})$ be a topological space.

(a) Show that X is Hausdorff if and only the diagonal

$$\Delta = \{(x, x) : x \in X\}$$

is closed in $X \times X$.

(b) Let Y be a Hausdorff space. Show that the graph $G(f) := \{(x, f(x)) : x \in X\}$ of a continuous function $f : X \longrightarrow Y$ is closed in the product space $X \times Y$.

Exercise 2. Let $\mathcal{C} = \{(x, e^x) : x \in \mathbb{R}\}$.

(a) Show that $\mathcal{C}$ is a closed set of $\mathbb{R}^2$.

(b) Show that $\pi_2 : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$ (the second projection) is not a closed map.

Exercise 3. Let X, Y be two topological spaces and $A \subseteq X, B \subseteq Y$.

(a) Show that

$$\operatorname{int}(A \times B) = \operatorname{int}(A) \times \operatorname{int}(B).$$

(b) Show that

$$\overline{A \times B} = \overline{A} \times \overline{B}.$$

(c) Show that

$$\text{Fr}(A \times B) = [\text{Fr}(A) \times \overline{B}] \cup [\overline{A} \times \text{Fr}(B)].$$

(d) Let $E = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1\}$. Find the frontier of E in the topological space $(\mathbb{R}^2, \mathcal{U})$.

(e) Show that $A \times B$ is dense in $X \times Y$ if and only if A is dense in X and B is dense in Y.

Exercise 4. A topological space $(X, \mathcal{T})$ is said to be *totally disconnected* if no subspace of X of cardinality greater than or equal to 2 is connected.

(a) Show that $(\mathbb{R}, \mathcal{LL})$ is totally disconnected, where $\mathcal{LL}$ is the lower limit topology on $\mathbb{R}$.

(b) Let $(X, \mathcal{T})$ be a T_0-space such that each $p \in X$ has a basis of clopen neighborhoods. Show that X is totally disconnected.

(c) Let $\{(X_i, \mathcal{T}_i) : i \in I\}$ be a family of topological spaces and $X = \prod_{i \in I} X_i$ be the product space.

Show that X is totally disconnected if and only if X_i is totally disconnected, for each $i \in I$.

Exercise 5. Let $\mathcal{CF}$ be the co-finite topology on $\mathbb{R}$.

(a) Show that $(\mathbb{R}, \mathcal{CF})$ is path-connected.

(b) Is there a continuous function from $(\mathbb{R}, \mathcal{CF})$ onto $(\mathbb{R}, \mathcal{LL})$?