King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 321 - Term 141
The Final Exam
Time allowed 180 minutes

Full name: ………………………………………………………………… ………

ID Number: …………………………………………………………………………

Question Number	**Full Mark**	**Your Mark**
Q1	**5**	
Q2	**8**	
Q3	**5**	
Q4	**10**	
Q5	**10**	
Q6	**10**	
Q7	**10**	
Q8	**5**	
Q9	**5**	
Q10	**8**	
Q11	**8**	
Q12	**8**	
Q13	**8**	
Total	**100**	

1. State the "Fixed Point Theorem" which gives sufficient conditions for an iteration $x_{n+1} = g(x_n)$ to converge to a fixed point.

2. Given the following table:

i	0	1	2	3
x_i	0	1	2	3
$f(x_i)$	2	3	10	29

Construct the appropriate table of divided differences and hence find a polynomial of degree 2 which interpolates $f\ at\ x_1, x_2\ and\ x_3$..

3. Show that Newton's method can be written as fixed point iteration

$$x_{n+1} = g(x_n)$$

for a suitable choice of $g(x)$.

4. Use the Bisection method to approximate the solution of $e^x - x = 2$ on $[1,2]$. (only 4 iterations required)

5. Use Gaussian elimination and three-digit chopping to this system

$$58.9x_1 + 0.03x_2 = 59.2,$$

$$-6.10x_1 + 5.31x_2 = 47.0,$$

With:

a) scaled partial pivoting

b) complete pivoting

6. Obtain factorization of the form $A = P^t LU$ for

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 2 & 3 \\ 2 & -1 & 4 \end{bmatrix}.$$

7. Construct the Jacobi and Gauss-Seidel methods for the following system:

$$\begin{bmatrix} 1 & 2 & -1 \\ 1 & 2 & 3 \\ 2 & -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 0 \end{bmatrix}.$$

Using $x^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, find the first two iterations if both methods apply to the system.

8. Consider the function

$$s(x) = \begin{cases} 0, & -1 \leq x \leq 0 \\ x^3 + ax^2 + bx + c, & 0 \leq x \leq 1. \end{cases}$$

Find a choice of the coefficients a, b, c such that $s(x)$ is a natural cubic spline function on $[-1,1]$.

9. For $f(x) = x^5 + \pi x^3 - \pi$, compute $f[0, 1, 2, 3, 4, 5]$.

10. Fit a straight line to the data $(1,3),(2,3),(3,4),(4,5),(5,5),(6,6),(7,6), and\ (8,7)$ using the Least Squares method.

11. Set up the system (no need to solve it) that fits the polynomial

$$f\ (x)\ =\ ax^3\ +\ bx$$

to the data $(-1,-7),(0,1),(1,6), and\ (3,-3)$.

12. Use RK4, Midpoint, and Modified Euler methods to approximate the missing entries (A, B, C) in the below table when this IVP:

$$y' = y - t^2 + 1, \qquad 0 \le t \le 2, \quad y(0) = 0.5$$

is solved.

t_i	**RK4**	**Midpoint**	**Modified Euler**
0.0	0.5	0.5	0.5
0.2	0.829293	0.828	0.826
0.4	1.214076	1.21136	1.20692
0.6	1.648922	1.6446592	1.6372424
0.8	2.127203	2.1212842	2.1102357
1	2.640823	2.6331668	2.6176876
1.2	**A**	3.1704634	3.1495789
1.4	3.73234	3.7211654	3.6936862
1.6	4.28341	4.2706218	4.2350972
1.8	4.815086	**B**	4.7556185
2	5.305363	5.2903695	**C**

13. Drive the Trapezoidal rule for approximating $\int_a^b f(x)dx$.

(Hint: use the linear Lagrange polynomial & the Weighted Mean Value Theorem).