King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 321 - Term 141
The 2^{nd} Exam
Time allowed 1 hour and 30 minutes

Full name: ....

ID Number: ....

Question Number	Full Mark	Your Mark
Q1	6	
Q2	3	
Q3	4	
Q4	6	
Q5	7	
Q6	6	
Q7	7	
Q8	6	
Q9	**5**	
Total	**50**	

1. **True** or **False** :

a) The Simpson's Rule is exact for all polynomials of degree 3 or less. (   )

b) The Trapezoidal Rule is exact for all polynomials of degree 2. (   )

c) For every IVP with $f(t, y)$ satisfies a Lipschitz condition in the variable y on the convex set D, the solution $y(t)$ is unique. (   )

d) As the step size h becomes smaller, more calculations are necessary and then more round-off error is expected. (   )

e) The precision of a quadrature formula is the largest positive integer n such that the formula is exact for x^k, for each $k = 0, 1, \dots, n$. (   )

f) Even if $f(t, y)$ of an IVP doesn't satisfy a Lipschitz condition, a unique solution may exist. (   )

2. Show that the following initial-value problem:

$$y' = t^{-10}(\sin 2\pi t - \pi t y), \qquad 1 \le t \le 10, \qquad y(1) = 2.$$

has a unique solution.

3. What is the degree of accuracy (precision) of the approximation of

$$\int_0^{3h} f(x)dx = \frac{3h}{4}[f(0) + 3f(2h)].$$

4. Set up the integral which determines $\int_{-1}^{1} xe^x \ dx$, accurate to 10^{-3}, using the Composite Simpson's Rule.

5. Consider the IVP

$$y' = ye^t + 1, \qquad y(0) = 0.5$$

(a) Use a value of $h = 1$ to approximate $y(1)$ by the fourth-order Runge-Kutta (RK) method, showing all equations and work.

(b) By how much would the truncation error in $y(1)$ decrease if one used a step size of $h = 0.25$? [Do **not** solve again for $h = 0.25$.]

$$k_1 = h \cdot f(t_i, y_i)$$
$$k_2 = h \cdot f(t_i + \frac{h}{2}, y_i + \frac{k_1}{2})$$
$$k_3 = h \cdot f(t_i + \frac{h}{2}, y_i + \frac{k_2}{2})$$
$$k_4 = h \cdot f(t_i + h, y_i + k_3)$$
$$y_{i+1} = y_i + \frac{1}{6} \cdot (k_1 + 2 \cdot k_2 + 2 \cdot k_3 + k_4)$$

6. Show that the error term of the following formula:

$$f'''(x_0) \approx \frac{f(x_0+2h) - 2f(x_0+h) + 2f(x_0-h) - f(x_0-2h)}{2h^3}$$

is $O(h^2)$.

7. Consider the IVP $y' = \frac{\cos(2ty)}{t^3}$, $1 \le t \le 2$, $y(1) = 0.5$.

a) Show that the IVP is well-posed problem.

b) Use Euler's method to approximate the solution in part (a) with $h = 0.5$.

8. Let $I = \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} cos^2 x \, dx$.

   a) Use the Trapezoidal's Rule to approximate I.
   b) Find a theoretical upper bound for the approximation error.

9. Drive the Runge-Kutta method of order two (using the 2^{nd} Taylor polynomial in two variables.)