King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 321 First Exam - Term 141
Time allowed 1 hour and 30 minutes

Full name: ..... ID Number: .....

Question Number	Full Mark	Your Mark
Q1	3	
Q2	4	
Q3	10	
Q4	4	
Q5	12	
Q6	12	
Q7	10	
Q8	5	
Total	**60**	

1. Consider the number $\pi = 3.14159265358979\ldots$.
If we use the approximation $\pi \approx 3.14$, what is the relative error? Express your answer using chopping to a decimal normalized floating-point representation with 5 significant digits.

2. Suppose that $fl(y)$ is 7-digit rounding approximation to y. Show that

$$\left|\frac{y - fl(y)}{y}\right| \leq 0.5 \times 10^{-6}.$$

3.

a) Find p_3 using the Bisection Method for $f(x) = 2x^2 - 1$ in the interval [0, 1]. (Hence an approximation to $p = \frac{1}{\sqrt{2}}$).

b) Find an estimate of the number of iterations *n* that would guarantee an error $|p_n - p|$ to be less than 10^{-4}.

4. The equation $x\cos x = x\sin x,$ has a root at $x = \frac{\pi}{4}$. Which (and why) of the following iteration process should be used to find this root?

$$x_{i+1} = x_i \tan x_i \quad \textbf{or} \quad x_{i+1} = x_i \cot x_i \ .$$

5.

a) Given the function values $f(0) = 1, f(1) = 3$, and $f(2) = 11$, find the quadratic interpolating polynomial $P_2(x)$.

b) If $|f'(x)| \leq 1, |f''(x)| \leq 1, \left|f'''\ \ (x)\right| \leq 1,\ and\ \left|f^{(4)}(x)\right| \leq 1,$ find an upper bound on the error for the Lagrange interpolating polynomial on the interval [0, 2].

6. For a function f, the divided differences are given in the following table:

$x_0 = 1$	$f[x_0]$		
$x_1 = 1.2$	4	$f[x_0, x_1]$	1
$x_2 = 1.4$	$f[x_2]$	3	

a) Determine the missing entries in the above table.

b) Find an approximation for $f(1.1)$ and $f(1.3)$. (one with Newton Forward-Divide Difference and the other one with Newton Backward-Divide Difference)

7. A natural cubic spline S of a function f on [0, 2] is defined by

$$S(x) = \begin{cases} S_0(x) = 1 + 2x - x^3, & for\ 0 \le x < 1, \\ S_1(x) = a + b(x-1) + c(x-1)^2 + d(x-1)^3, & for\ 1 \le x \le 2. \end{cases}$$

Find $a, b, c,$ and d and find an approximation to $f(1.1)$.

8. If $L_k(x)$ is the Lagrange interpolating polynomials. i.e.

$$L_k(x) = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{x - x_i}{x_k - x_i}.$$

Show that $\sum_{k=1}^{n} L_k(x) = 1$, for any real x, integer n, and any set of distinct points $x_1, \ldots, x_n$.