King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Instructor: Khaled Furati

MATH 311 - Final Exam - Term 141

Duration: 150 minutes

Student Name:

Question Number	**Points**	**Maximum Points**
1		8
2		6
3		6
4		6
5		4
6		5
7		5
8		5
9		5
Total		50

1. State (without explanation) whether the following statements are true or false.

   (a) If (x_n) is bounded convergent sequence then $\overline{\lim}(x_n) = \underline{\lim}(x_n)$.

   (b) If $f : A \to \mathbb{R}$ is a uniformly continuous function then f is a Lipschitz function on A.

   (c) If a series is convergent in $\mathbb{R}$ then any series obtained from it by grouping is also convergent.

   (d) If a series is convergent in $\mathbb{R}$ then any series obtained from it by rearrangement is also convergent.

2. Prove the following: A number $c \in \mathbb{R}$ is a cluster point of a subset A of $\mathbb{R}$ <u>if and only if</u> there exists a sequence (a_n) in A such that $\lim(a_n) = c$ and $a_n \neq c$ <u>for all $n \in \mathbb{N}$</u>.

3. Let f be continuous on $[a, b]$ and differentiable on (a, b). Show that if $f'(x) = 0$ for $x \in (a, b)$ then f is constant on $[a, b]$.

4. State without proof the following.

   (a) The <u>first form</u> and <u>second form</u> of the Fundamental Theorem of Calculus.

   (b) Lebesgue's integrability Criterion.

5. Prove that if a series in $\mathbb{R}$ is absolutely convergent then it is convergent.

6. Test the following series for convergence and for absolute convergence.

$$\sum_{n=1}^{\infty} \frac{(-1)^n \, 2^n}{n^2}.$$

7. Establish the convergence or divergence of the series

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{2n^2 - 1}.$$

8. Establish the convergence or divergence of the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}.$$

9. Use Abel's test to show the convergence of the series

$$\sum_{n=1}^{\infty} n^{\frac{1}{n} - 2}.$$