King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
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MATH 311 - Exam 1 - Term 141

Duration: 90 minutes

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1. State without proof the following.
   (a) Completeness property of $\mathbb{R}$.
   (b) Monotone Convergence Theorem.
2. Define the following.
   (a) $\sup S$, where S is a nonempty bounded above subset of $\mathbb{R}$.
   (b) Cauchy sequence.
3. Prove that if $a \in \mathbb{R}$ is such that $0 \leq a \leq \epsilon$ for every $\epsilon > 0$, then $a = 0$.
4. If $a < x < b$ and $a < y < b$, show that $|x-y| < b-a$. Interpret this geometrically.
5. Let (x_n) be the sequence given by

$$x_1 = 1, \qquad x_2 = 3, \qquad x_n := \frac{1}{3}\, x_{n-1} + \frac{2}{3}\, x_{n-2}, \qquad n > 2.$$

   Show that (x_n) is convergent.