Math 302

Quiz 1

18/ 9/ 2014

Name: ID #

Problem 1 (5 points): Let

$$X = \{a =< a_1, a_2, a_2 - a_1, a_1 - a_2 > \ / \ a_1, a_2 \in \mathbb{R}\}$$

1) Show that X is a subspace of $\mathbb{R}^4$
2) Find a basis and $\dim X$.

Sol. (1) Let $a, b \in X$. Then

- $a + b = < a_1, a_2, a_2 - a_1, a_1 - a_2 > + < b_1, b_2, b_2 - b_1, b_1 - b_2 >$

  $= < a_1 + b_1, a_2 + b_2, (a_2 + b_2) - (a_1 + b_1), (a_1 + b_1) - (a_2 + b_2) >$

  $= < c_1, c_2, c_2 - c_1, c_1 - c_2) \in X$

- let $k \in \mathbb{R}$, then $ka = < ka_1, ka_2, ka_2 - ka_1, ka_1 - ka_2 >$

  $= < d_1, d_2, d_2 - d_1, d_1 - d_2 > \in X$

$\therefore$ X is a subspace. of $\mathbb{R}^4$.

- $a \in X \Leftrightarrow a = < a_1, a_2, a_2 - a_1, a_1 - a_2 >$

  $= a_1 < 1, 0, -1, 1 > + a_2 < 0, 1, 1, -1 >$

2) A basis is $B = \{ < 1, 0, -1, 1 >, < 0, 1, 1, -1 > \}$

$\dim X = 2$

Problem 2 (5 points):
Use Gass-Jordan elimination method to find the solution set of the system

$$\begin{cases} x_1 - 2x_2 + x_3 = 1 \\ 3x_1 \quad\;\; - x_3 = 4 \end{cases}$$

Sol.

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 3 & 0 & -1 & 4 \end{array}\right) \xrightarrow{R_2-3R_1} \left(\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 6 & -4 & 1 \end{array}\right) \xrightarrow{R_2/6}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 1 & -\frac{2}{3} & \frac{1}{6} \end{array}\right) \xrightarrow{R_1+2R_1} \left(\begin{array}{ccc|c} 1 & 0 & -1/3 & 4/3 \\ 0 & 1 & -2/3 & 1/6 \end{array}\right)$$

Reduced (equivalent) system

$$\begin{cases} x_1 - 1/3\, x_3 = 4/3 \\ x_2 \quad - 2/3\, x_3 = 1/6 \end{cases} \iff \begin{cases} x_1 = 4/3 + \frac{1}{3}x_3 \\ x_2 = 1/6 + \frac{2}{3}x_3 \end{cases}$$

~~If~~ The solution set is

$$X = \left\{ \left< \frac{4}{3}, \frac{1}{6}, 0 \right> + t \left< \frac{1}{3}, \frac{1}{6}, 1 \right>, \; t \in \mathbb{R} \right\}$$

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Problem 1 (5 points): Let

$$X = \{a =< a_1, a_1, a_3, a_1 - a_3 > \ / \ a_1, a_3 \in \mathbb{R}\}$$

1) Show that X is a subspace of $\mathbb{R}^4$
2) Find a basis and $\dim X$.

Sol.

1) let $a, b \in X$. Then

- $a+b = < a_1, a_1, a_3, a_1 - a_3 > + < b_1, b_1, b_3, b_1 - b_3 >$
  $= < a_1 + b_1, a_1 + b_1, a_3 + b_3, (a_1 + b_1) - (a_3 + b_3) >$
  $= < c_1, c_1, c_3, c_1 - c_3 > \in X$.
- If $k \in \mathbb{R}$, then $ka = < ka_1, ka_1, ka_3, ka_1 - ka_3 >$
  $= < d_1, d_1, d_3, d_1 - d_3 > \in X$

So X is a subspace of $\mathbb{R}^4$.

(2) $a \in X \Leftrightarrow a = < a_1, a_1, a_3, a_1 - a_3 >$
$= a_1 < 1, 1, 0, +1 > + a_3 < 0, 0, 1, -1 >$

A basis is $B = \{ < 1, 1, 0, 1 >, < 0, 0, 1, -1 > \}$

$\dim X = 2$.

Problem 2 (5 points):
Use Gass-Jordan elimination method to find the solution set of the system

$$\begin{cases} x_1 + 2x_2 - x_3 = 1 \\ 3x_1 - x_2 \qquad = 5 \end{cases}$$

Sol.

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 3 & -1 & 0 & 5 \end{array}\right) \xrightarrow{R_2 - 3R_1} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -7 & 3 & 2 \end{array}\right) \xrightarrow{R_2/-7}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & -\frac{3}{7} & -2/7 \end{array}\right) \xrightarrow{R_1 - 2R_2} \left(\begin{array}{ccc|c} 1 & 0 & -1/7 & \frac{11}{7} \\ 0 & 1 & -3/7 & -2/7 \end{array}\right)$$

Reduced (equivalent) system

$$\begin{cases} x_1 - \frac{1}{7}x_3 = \frac{11}{7} \\ x_2 - \frac{3}{7}x_3 = -\frac{2}{7} \end{cases} \iff \begin{cases} x_1 = \frac{11}{7} + \frac{1}{7}x_3 \\ x_2 = -\frac{2}{7} + \frac{3}{7}x_3 \end{cases}$$

The solution set is

$$X = \left\{ < \frac{11}{7}, -\frac{2}{7}, 0> + t < \frac{1}{7}, \frac{3}{7}, 1>, \ t \in \mathbb{R} \right\}$$