King Fahd University of Petroleum and Minerals

Department of Mathematics and Statistics

Math 302 Final Exam

Semester (141) Dec. 30, 2014 at 8:00-11:00 AM

Name: ....

I.D: .... Section: ....   Serial: ....

Question	Points
1	/14
2	/14
3	/14
4	/14
5	/14
6	/14
7	/14
8	/14
9	/14
10	/14
Total	/140

a. Let $A = \begin{pmatrix} -1 & 3 & 0 \\ 3 & -2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$, use the Guaussian elimination method to **find** A^{-1}.

b. Use (a) to **solve** the system

$$\begin{cases} -x_1 + 3x_2 & = 1 \\ 3x_1 - 2x_2 + x_3 = 0 \\ x_2 + 2x_3 = 0 \end{cases}$$

Let $A = \begin{pmatrix} 1 & 0 & 7 \\ 0 & 1 & 0 \\ 7 & 0 & 1 \end{pmatrix}$

a) Find all **eigenvalues** of A and the corresponding **eigenvectors.**

b) **Find** a matrix P that diagonalizes A.

c) If $D = P^{-1}AP$, **find** D^4 and write a **formula** to evaluate A^4.(Do not calculate A^4)

Question 3   (14 points)

Find $\oint_C (-xy)dx + (e^{-y^2})dy,$ where C is the positively oriented closed path

consisting of the quarter circle joining the points $(2,0), (0,2)$, the line segment

joining the points $(0,2), (-2,0)$, and the line segment joining the points

$(-2,0), (2,0)$.

Question 4   (14 points)

Let $\mathbf{F}(x, y, z) = xy^2\mathbf{i} + yx^2\mathbf{j} + xy\mathbf{k}$ be a field and S be the surface enclosing the volume between the paraboloids $z = 4 - x^2 - y^2$ and $z = 1 - x^2 - y^2$ and above the $x - $ axis.

Find the flux $\iint_S \mathbf{F}.\mathbf{n}\, dS$, where $\mathbf{n}$ is the outer unit normal vector to S.

Question 5 (14 points)

a. Let $f(z) = x^2 - x + y + i(y^2 - 5y - x)$. **Show** that $f(z)$ is differentiable at

$z_0 = 3 + 5i$ and **find** $f'(z_0)$.

b. If $f(z) = e^x \cos y + iv(x, y)$ is an analytic function for any z.

Find $v(x, y)$ and **write** $f(z)$ in terms of z.

Question 6   (14 points)

a. If $\ln(-ei) = a + ib$, **find** a and b.

b. **Solve:** $2\cosh z + \sinh z = 2$.

c. **Show** that $i\pi$ is a period for the periodic function $f(z) = \tanh z$.

Question 7

Use Cauchy's integral formulas to **evaluate**

$$I = \oint_C \frac{\sin z}{z^4 + z^2} dz \text{ , where } C \text{ is the circle } |z - i| = \frac{3}{2}.$$

Let $f(z) = \dfrac{e^{-iz^2} - 1}{z^5}$.

a. **Find** the Laurent series in $|z| > 0$ and **classify** its pole.

b. Use the result in (a) to **find** $\oint_C f(z)dz$, where C is given in the adjacent figure.

Use the residue theorem to evaluate

$$\oint_C \frac{e^z - 1}{z^2(z-1)^2}\, dz, \text{ where } C \text{ is the circle } |z| = 2.$$

Compute $\int_{-\infty}^{+\infty} \frac{2x^2-1}{x^4+5x^2+4}\,dx$.