King Fahd University of Petroleum and Minerals

Department of Mathematics and Statistics

Math 302 Exam I

Semester (141) Oct. 16, 2014 at 5:45-7:45 PM

Name: ....

I.D: .... Section: .... Serial: ....

Question	Points
1	/20
2	/10
3	/25
4	/25
5	/20
Total	/100

Question 1 (20 points)

Consider the vectors $u_1 =< 0,1,0 >$, $u_2 =< 0,1,1 >$, $u_3 = < 1,1,1 >$ of $\mathbb{R}^3$.

a. Show that u_1, u_2, u_3 are linearly independent.

b. Write $v =< a, b, c >$ as a linear combination of u_1, u_2, u_3.

c. Does $\{u_1, u_2, u_3\}$ form a basis of $\mathbb{R}^3$. (why?)

<u>Question 2</u> <u>(10 points)</u>

Let $W = \{(x, y) \in \mathbb{R}^2 : 2x - y = 0\}$.

a. Show that W is a subspace.

b. Find a basis and the dimension of W.

Question 3 (25 points)

Consider the following system

$$\begin{cases} x_1 - 2x_2 + x_3 = 2 \\ 3x_1 - x_2 + 2x_3 = 5 \\ 2x_1 + x_2 + x_3 = \alpha \end{cases}$$

a. Find all values of α for which the system is consistent.

b. If the above system is written as $AX = (2 \quad 5 \quad \alpha)^T$, find $Rank(A)$.

<u>Question 4</u> <u>(25 points)</u>

Given the matrix $A = \begin{pmatrix} 5 & -1 & 0 \\ 0 & -5 & 9 \\ 5 & -1 & 0 \end{pmatrix}$.

a. Find the eigenvalues of A.

b. Find a corresponding eigenvector to the largest eigenvalue (only).

<u>Question 5</u> <u>(20 points)</u>

Given the symmetric matrix $A = \begin{pmatrix} 0 & a & 1 \\ a & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$.

a. Find the eigenvalues of A and check that they are real, for any $a \in \mathbb{R}$.

b. Are there values of a, for which the matrix A is orthogonal?