Math 302

Quiz 2

28/ 10/ 2014

Name: **ID #**

Problem 1 (5 points): Show that $A = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}$ is not diagonalizable

Sol. Eigenvalues

$$\begin{vmatrix} -\lambda & 1 \\ -1 & 2-\lambda \end{vmatrix} = \lambda^2 - 2\lambda + 1 = (\lambda - 1)^2 = 0$$

$\Rightarrow$ $\lambda = 1$ is a repeated eigenvalue.

Eigenvectors

$$\left(\begin{array}{cc|c} -1 & 1 & 0 \\ -1 & 1 & 0 \end{array}\right) \xrightarrow[R_2 - R_1]{-R_1} \left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$\Rightarrow$ $x_1 = x_2$ $\Rightarrow$ $X = x_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow E = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is an eigenvector.

We have only eigenvector for a repeated eigenvalue.

So it is not diagonalizable.

Problem 2 (2.5 points):
Find the length of the curve C given by $r(t) = \langle 3t, t^2\sqrt{3}, \frac{2}{3}t^3 \rangle$, $0 \le t \le 1$.

Sol. $r'(t) = < 3, 2t\sqrt{3}, 2t^2 >$

$$\| r'(t) \| = \sqrt{9 + 12t^2 + 4t^4} = \sqrt{(2t^2+3)^2} = 2t^2 + 3$$

$\Rightarrow$ The length is

$$\ell = \int_0^1 \| r'(t) \| \, dt = \int_0^1 (2t^2+3)\, dt$$

$$= \frac{2}{3}t^3 + 3t \Big|_0^1 = \frac{2}{3} + 3 = \frac{11}{3}$$

Problem 3 (2.5 points):
Find $D_u F(2, 4, -1)$,if $F(x, y, z) = \frac{x^2 - y^2}{z^2}$ and $u = \langle 1, -2, 1 \rangle$

$\| u \| = \sqrt{6} \Rightarrow v = < \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}} >$

is a unit vector.

$$D_u F(2,4,-1) = \nabla F(2,4,-1) \cdot v$$

$$\nabla F = < \frac{2x}{z^2}, \frac{-2y}{z^2}, \frac{-2(x^2-y^2)}{z^3} >$$

$$\nabla F(2,4,-1) = < 4, -8, -24 >$$

$$D_u F(2,4,-1) = \frac{4}{\sqrt{6}} + \frac{16}{\sqrt{6}} - \frac{24}{\sqrt{6}} = -\frac{4}{\sqrt{6}}.$$

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Problem 1 (5 points): Show that $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{pmatrix}$ is diagonalizable

Sol. Eigenvalues

$$\begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & 3 \\ 0 & 0 & 2-\lambda \end{vmatrix} = (1-\lambda)^2(2-\lambda) = 0$$

Eigenvalues are $\lambda = 2$ and $\lambda = 1$ repeated.

Eigenvectors for $\lambda = 1$

$$\left(\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \xrightarrow[R_3 - R_1]{R_2 - 3R_1} \left(\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\Rightarrow x_3 = 0 \Rightarrow X = \begin{pmatrix} x_1 \\ x_2 \\ 0 \end{pmatrix} = x_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

So, we have two independent eigenvectors

$$E_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad E_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$\lambda = 2$ gives another independent eigenvectors.

So A is diagonalizable.

Problem 2 (2.5 points):
Find the length of the curve C given by $r(t) = \langle \frac{1}{2}t^2, \frac{2\sqrt{2}}{3}t^{3/2}, t \rangle$, $0 \le t \le 2$.

$$r'(t) = \langle t, \sqrt{2}\, t^{\frac{1}{2}}, 1 \rangle$$

$$\| r'(t) \| = \sqrt{t^2 + 2t + 1} = \sqrt{(t+1)^2} = t + 1$$

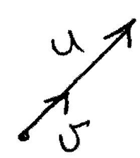
The length is

$$\ell = \int_0^2 \| r'(t) \| \, dt = \int_0^2 (t+1)\, dt$$

$$= \frac{t^2}{2} + t \Big|_0^2 = 2 + 2 = 4.$$

Problem 3 (2.5 points):
Find $D_u F(2, 0, -1)$, if $F(x, y, z) = \frac{x^2 y^2}{z}$ and $u = \langle 1, -2, 1 \rangle$

$\| u \| = \sqrt{6} \implies v = \langle \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \rangle$ is a unit vector in the direction of u



$$\nabla F = \langle \frac{2xy^2}{z}, \frac{2x^2 y}{z}, -\frac{x^2 y^2}{z^2} \rangle$$

$$\nabla F(2, 0, 1) = \langle 0, 0, 0 \rangle$$

$$D_u F(2, 0, -1) = 0.$$