King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
(Math 260)

Code: C

Final Exam
Term 141
Tuesday, Jan 06, 2015
Building No. 10
Net Time Allowed: 180 minutes
07:00PM to 10:00PM

Name:		**ID:**	

(SHOW ALL YOUR STEPS AND WORK)

PART-I	**WRITTEN - PART**	
Q		**Points**
1		/15
2		/15
3		/15
4		/15
5		/15
6		/15
Total		**/90**

PART-II	**MCQ - PART**					
Q	**Answers**					**Points**
1	a	b	c	d	e	/6
2	a	b	c	d	e	/6
3	a	b	c	d	e	/6
4	a	b	c	d	e	/6
5	a	b	c	d	e	/6
6	a	b	c	d	e	/6
7	a	b	c	d	e	/6
8	a	b	c	d	e	/6
9	a	b	c	d	e	/6
10	a	b	c	d	e	/6
11	a	b	c	d	e	/6
12	a	b	c	d	e	/6
13	a	b	c	d	e	/6
14	a	b	c	d	e	/6
15	a	b	c	d	e	/6
Total						**/90**
Grand Total						**/180**

WRITTEN PART

1) Set up an appropriate form of a particular solution y_p of the following differential equations (Do not DETERMINE THE VALUES OF THE COEFFICIENTS):

$$y'' - y' - 2y = 6x + 6e^{-x}$$

2) Determine whether the matrix $A = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{bmatrix}$ is diagonalizable

or not.

3) Let $x_{k+1} = A x_k$, whe $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ and $x_0 = \begin{bmatrix} \frac{1}{5^9} \\ \frac{1}{5^9} \end{bmatrix}$. Find x_{10}

Hint: the eigenvalues of A are 0 and 5.

4) Transform the differtial equation $y''' + 5\,y'' - 8y = 2e^{t}$ to an equivalent system of first-order differential equations and write the system in matrix form.

5) Find a general solution of the system

$$X' = \begin{bmatrix} 1 & 3 \\ -3 & 1 \end{bmatrix} X$$

6) Find a general solution of the system

$$Y' = AY \quad \text{where} \quad A = \begin{bmatrix} 1 & -4 & 0 \\ 4 & 9 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad \text{and } Y = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

MCQ PART

1) Which of the following subsets of $\mathcal{R}^3$ is a subspace of $\mathcal{R}^3$

A) The set of all vectors (x, y, z) such that $x^2 = y^2$
B) The set of all vectors (x, y, z) such that $x + y - z^2 = 0$
C) The set of all vectors (x, y, z) such that $x + y = z + 1$
D) The set of all vectors (x, y, z) such that $x - yz = 0$
E) The set of all vectors (x, y, z) such that $3x + 4y = 5z$

2) Let $u = (1, 2, 3)$, $v = (3, 0, 1)$, $w = (-5, 8, 9)$ which one of the following statements is TRUE?

A) w is a linear combination of u and v.
B) $\{v, w\}$ spans $\mathcal{R}^3$
C) w is in the span of (1, 0, 0) and (0, 1, 0)
D) $\{u, w\}$ spans $\mathcal{R}^3$
E) $\{u, v\}$ is linearly dependent

3) Let $A = \begin{bmatrix} 1 & 5 & 1 \\ 0 & 2 & 1 \\ 2 & 5 & 0 \end{bmatrix}$. Then the element in the first row and first column of A^{-1} is:

A) 1

B) 5

C) 3

D) -3

E) -5

4) The Wronskian of the functions

$f_1(x) = x$, $f_2(x) = x + x^2$, and $f_3(x) = 2x - x^2$ equals to:

A) x

B) -x

C) 0

D) +1

E) -1

5) If $\begin{bmatrix} a & b & c \\ x & y & z \end{bmatrix}$ is the reduced row echelon form of $\begin{bmatrix} 5 & 2 & 18 \\ 4 & 1 & 12 \end{bmatrix}$,

then $b + c =$

A) 1
B) 7
C) 6
D) 0
E) 2

6) If the DE $\left(kxy^3 + \cos y\right) dx + \left(3x^2 y^2 - x\sin y\right) dy = 0$ is exact, then $k =$

A) 1
B) 3
C) 2
D) 4
E) 6

7) If y_p is a particular solution of the IVP

$y'' - 2y' - 3y = 0, \quad y(0) = 0, \quad y'(0) = -4,$

then $y_p(1) =$

A) $e - e^3$
B) $e^{-1} - e^{-3}$

C) $e^{-1} - e^3$

D) $e - e^{-3}$

E) $e + e^3$

8) If A and B are 2×2 matrices such that $\det A = 6$ and $\det B = 3$, then

$\det(2AB^{-1})$

A) 12
B) 24

C) 6
D) 18
E) 8

9) Which of the following differntial equation has $y = c_1e^{3x} + c_2xe^{3x}$ as general solution?

A) $xy' - (1 + 3x)\, y = 0$
B) $y'' - 6\, y' + 9y = 0$
C) $y'' - 3\, y' + 6y = 0$
D) $y'' + 3\, y' + 9y = 0$
E) $y' - 3y = 0$

10) If $y = ae^{3x} + be^{-5x}$ is a solution of the IVP:

$y'' + 2y' - 15y = 0, \quad y(0) = 40, \quad y'(0) = -16$

then $a - b =$

A) 0
B) 4
C) 8
D) 6
E) 2

11) The Bernoulli DE $y' + \frac{1}{x} y = 3x^2 y^3$ can be written as first order linear DE as follows:

A) $v' + \frac{2}{x} v = -6x^2$

B) $v' + \frac{1}{2x} v = -\frac{3}{2} x^2 v^{-3}$

C) $v' - \frac{2}{x} v = -6x^2$

D) $v' - \frac{2}{x} v = -\frac{3}{2} x^2 v^{-3}$

E) $v' - \frac{1}{2x} v = -\frac{3}{2} x^2 v^{-3}$

12) If $2y'\sqrt{x} = -e^y$ and $y(1) = 0$, then $y(e^2) =$

A) 0
B) -1
C) 1
D) -2
E) 2

13) If $\frac{dy}{dx} = \frac{x}{\sqrt{x^2 + 16}}$ and $y(0) = 2$, then $y(3) =$

A) 1
B) 2
C) 3
D) 5
E) 4

14) The value of y when $x = 2$ that satisfies the first order IVP

$$y' = 4x^2 - \frac{y}{x}, \qquad y(1) = 5$$

is equal to

A) 0
B) 7
C) -5
D) 10
E) 3

15) Which of the following sets is a basis for $\mathcal{R}^3$

A) $\{ (2, 1, -1), (1, 2, -1), (1, 1, 2), (1, 1, -2) \}$

B) $\{ (2, -1, 1), (1, -2, 1) \}$

C) $\{ (2, 1, 1), (4, 2, 2), (0, 1, 0) \}$

D) $\{ (2, 1, 1), (4, 1, 1), (6, 2, 2) \}$

E) $\{ (2, 1, 1), (1, 2, 1), (0, 0, 1) \}$