King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
(Math 260)

Second Major Exam
Term 141
Thursday, November 20, 2014
Net Time Allowed: 90 minutes

Name:	
ID:	
Section No:	
Instructor's Name:	

(Show all your steps and work)

Question #	Marks
1	10
2	10
3	10
4	8
5	10
6	10
7	12
8	10
Total	**/80**

(1) (a) Find the inverse of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ [5 points]

(b) Use the answer in part (a) to find a $3X3$ matrix X such that [5 points]

$$AX = C, \quad where \quad C = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

(2) Use Cramer's rule to solve the system. [10 points]

$$\begin{aligned} 2x_1 + 6x_2 + x_3 &= 7 \\ x_1 + 2x_2 - x_3 &= -1 \\ 5x_1 + 7x_2 - 4x_3 &= 9 \end{aligned}$$

(3) a) Determine whether or not the vectors $(1,0,0,3)$, $(0,1,-2,0)$ *and* $(0,-1,1,1)$ are linearly independent. [5 points]

b) Express (if possible) the vector $W=(2,3,4)$ as a linear combination of the vectors $V_1=(1,2,0)$, $V_2=(0,2,3)$, $V_3=(1,2,1)$ [5 points]

(4) Find a second-order differential equation with constant coefficients whose general solution is $y(x) = e^{x}\left(c_1 e^{x\sqrt{2}} + c_2 e^{-x\sqrt{2}}\right)$. [8 points]

(5) Determine whether or not W is a subspace of $\mathbb{R}^3$ if: [5 points]

a) W is the set of all $(x, y, z) \varepsilon \mathbb{R}^3$ such that $xy(z+1)=0$

b) W is the set of all $(x, y, z) \varepsilon \mathbb{R}^3$ such that $2x+3y-3z=0$ [5 points]

(6) Find a basis for the solution space of the homogeneous system [10 points]

$$x_1 - 4x_2 - 3x_3 - 7x_4 = 0$$
$$2x_1 - x_2 + x_3 + 7x_4 = 0$$
$$x_1 + 2x_2 + 3x_3 + 11x_4 = 0$$

(7) a) Determine whether or not the solutions $y_1 = x,\ y_2 = x^{-2},\ y_3 = x^{-2}\ln x$ of the differential equation $x^3 y''' + 6x^2 y'' + 4xy' - 4y = 0$ are linearly independent.

[6 points]

b) Use part (a) to find a particular solution of the differential equation: [6 points]

$x^3 y''' + 6x^2 y'' + 4xy' - 4y = 0$ that satisfies the initial conditions

$y(1) = 1, y'(1) = 5, y''(1) = -11$

(8) Find a general solution of the differential equation [10 points]

$$\left(D^2+4\right)\left(D^4+4D^3+8D^2+16D+16\right)y=0$$

knowing that $y = xe^{-2x}$ is a solution of the equation.