

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 232 Final Exam (Term 141)

Name : ID #..... Serial #:

Question 1: Fill in the blanks

1. The power set P of the set $\{\phi\}$ is $P=$

2. The set $\{[n, n + 1) : n \in \mathbb{Z}\}$ is a partition of the well-known set:

3. $\cup_{k \in \mathbb{N}} [0, \frac{k}{k+1}] =$

4. If $A = \{\pi\}$ and $B = \{0\}$, then $A \times B \times B \times A =$

5. For any statements P and Q , the negation of the implication $P \implies Q$ is

6. The least element, if exists, of the set $\cap_{n \in \mathbb{N}} (-\frac{1}{n}, 1]$

7. A function f is injective (one to one) if

8. An example of a bijection $f : \mathbb{R} \rightarrow (0, 1)$ is $f(x) =$

9. The set $\mathbb{Z}_5 =$

10. A binary operation $*$ on a set A is

Question 2: Let R be an equivalence relation on a nonempty set A . Show that for every $a, b \in A$, if aRb , then $[a] = [b]$.

Question 3: Let $a \in \mathbb{Z}$ and $n \in \mathbb{N}$ with $n \geq 2$. Use a direct proof to show that

$$a \equiv n \pmod{n} \implies a \equiv 0 \pmod{n}$$

Question 4: Prove by contradiction that there do not exist positive integers m and n such that $m^2 - n^2 = 1$.

Question 5: Let m be a fixed integer. Show that the set

$$A = \{x \in \mathbb{Z} : x \geq m\}$$

is denumerable by using a bijection.

Question 6: Let $\{x_i\}_{i \in \mathbb{N}}$ be a sequence of real numbers. Use mathematical induction to prove that

$$|\sum_{i=1}^n x_i| \leq \sum_{i=1}^n |x_i|, \quad \forall n \geq 2$$

Question 7: Let $A = \mathbb{R} - \{0\}$, and $*$ be the usual multiplication, that is a binary operation on A . Prove/Disprove that the algebraic structure $(A, *)$ is an abelian group.