Math 232 Exam II

Sr. Num.: ID. Num.: Name:

Question 1: Let a, b, $c \in R^+$. Prove that if $|x-3| < a$ and $|x-3| < \frac{b}{c}$, then

$$|x^2 - 7x + 12| < \frac{(a+4)b}{c}$$

Question 2: Let $a \in Z$ such that $a \equiv 2 \pmod 5$. Show that $a^3 \equiv 8 \pmod 5$.

Question 3: Prove that there exists no $n \in Z$ such that $n^2 + n + 1$ is even.

Question 4: Prove or disprove

(1) The sum of a rational number and an irrational number is irrational.
(2) The sum of two irrational numbers is irrational.

Question 5: State and prove the general principle of mathematical induction.

Question 6: Use induction to prove that for all $n \in N$,

$$1 + 3 + 5 + ... + (2n-1) = n^2$$

Question 7: Let A be a nonempty set. Define the following terms:

(1) a relation on A
(2) an equivalence relation on A
(3) an equivalence class $[a]$, for $a \in A$

Question 8: Let $H = \{2^m : m \in Z\}$. A relation R on Q^+ is defined by $a\ R\ b$ if $\frac{a}{b} \in H$.

(1) Show that R is an equivalence relation.
(2) Describe in a set notation the elements of [3].