KFUPM MATH201 Quiz#3 Time:20min. Sec.21

Q1 Find the distance from the plane $x + 2y + 6z = 1$ to th plane $x + 2y + 6z = 10$.

The point $P(1, 0, 0)$ is on the first plane and $S(10, 0, 0)$ is a point on the second plane $\Rightarrow \overrightarrow{PS} = 9\mathbf{i}$, and $\mathbf{n} = \mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$ is normal to the first plane $\Rightarrow$ the distance from S to the first plane is $d = \left| \overrightarrow{PS} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right|$ $= \left| \frac{9}{\sqrt{1+4+36}} \right| = \frac{9}{\sqrt{41}}$, which is also the distance between the planes.

Q2. Find the limit $\lim_{(x,y)\to(0,0)} \frac{xy}{|xy|}$ if it exists.

Path $y = x \to \lim_{(x,y)\to(0,0)} \frac{xy}{|xy|} = \lim_{x\to 0} \frac{x^2}{|x^2|} = 1$

Path $y = -x \to \lim_{(x,y)\to(0,0)} \frac{xy}{|xy|} = \lim_{x\to 0} \frac{-x^2}{|x^2|} = -1$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{|xy|} \ DNE$$

Q3. $z = \tan^{-1}\left(\frac{x}{y}\right)$, $x = u\cos(v)$, $y = u\sin(v)$. Express $\frac{\partial z}{\partial u}$ in terms of u and v

8. (a) $\frac{\partial z}{\partial u} = \left[\frac{\left(\frac{1}{y}\right)}{\left(\frac{x}{y}\right)^2 + 1} \right] \cos v + \left[\frac{\left(\frac{-x}{y^2}\right)}{\left(\frac{x}{y}\right)^2 + 1} \right] \sin v = \frac{y \cos v}{x^2 + y^2} - \frac{x \sin v}{x^2 + y^2} = \frac{(u \sin v)(\cos v) - (u \cos v)(\sin v)}{u^2} = 0;$