Q1. Describe the set of all points in a space equidistant from the origin and the point (0, 2, 0) with single equation.

Let the points $P(x, y, z)$, *then*

the distance between P and the origin
$$= \text{the distance between P and the ponit}(0, 2, 0)$$

$$\sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2} = \sqrt{(x-0)^2 + (y-2)^2 + (z-0)^2}$$

$$\sqrt{x^2 + y^2 + z^2} = \sqrt{x^2 + y^2 - 4y + 4 + z^2}$$

$$x^2 + y^2 + z^2 = x^2 + y^2 - 4y + 4 + z^2$$

$$\rightarrow y = 1$$

Q2. Using the definition of the projection of **u** onto **v,** show by calculation that

$$(u - \text{proj}_v u) \cdot \text{proj}_v u = 0.$$

$$\text{proj}_v u = \frac{u \cdot v}{|v|^2} v \rightarrow (u - \text{proj}_v u) \cdot \text{proj}_v u = \left(u - \frac{u \cdot v}{|v|^2} v\right) \cdot \frac{u \cdot v}{|v|^2} v$$

$$= u \cdot \frac{u \cdot v}{|v|^2} v - \left(\frac{u \cdot v}{|v|^2} v\right) \cdot \left(\frac{u \cdot v}{|v|^2} v\right) = \left(\frac{u \cdot v}{|v|^2}\right)(u \cdot v) - \left(\frac{u \cdot v}{|v|^2}\right)^2 (v \cdot v)$$

$$= \frac{(u \cdot v)^2}{|v|^2} - \frac{(u \cdot v)^2}{|v|^4} |v|^2 = \frac{(u \cdot v)^2}{|v|^2} - \frac{(u \cdot v)^2}{|v|^2} = 0$$

Q3. Find a unit vector perpendicular to the plane of

$$P(1, -1, 2), Q(2, 0, -1), \text{ and } R(0, 2, 1).$$

$$\vec{PQ} = \langle 1, 1, -3 \rangle, \ \vec{PR} = \langle -1, 3, -1 \rangle$$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ 1 & 1 & -3 \\ -1 & 3 & -1 \end{vmatrix} = 8i + 4j + 4k$$

$$|\vec{PQ} \times \vec{PR}| = \sqrt{64 + 16 + 16} = 4\sqrt{6}$$

$$u = \frac{\vec{PQ} \times \vec{PR}}{|\vec{PQ} \times \vec{PR}|} = \frac{1}{\sqrt{6}}(2i + j + k)$$