KFUPM MATH201 **Time: 25min** Quiz#1 **Sec.21**

Name: ____________________ Sr#: ____

Q1. Let C be the curve defined by the parametric equations

$$x = -2 + 4cos(t),\ y = 1 + 3sin(t),\ 0 \le t \le \pi$$

a) Find Cartesian equation for C
b) Sketch the graph and indicate on it the direction in which C traced.
c) Find the area of the surface generated by revolving C about the *x-axis*.

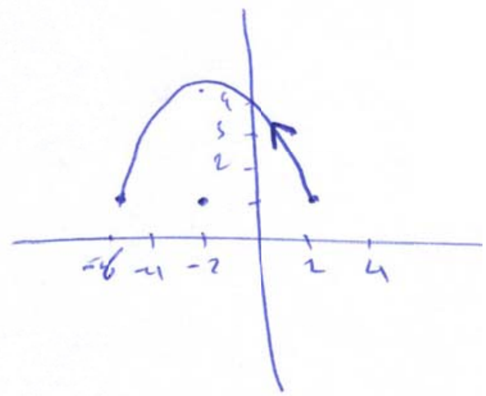
a) $\left(\frac{x+2}{4}\right)^2 + \frac{(y-1)^2}{9} = \cos^2 t + \sin^2 t = 1$

$$\frac{(x+2)^2}{16} + \frac{(y-1)^2}{9} = 1$$

b) $t = 0 \to (2, 1)$

$t = \frac{\pi}{2} \to (-2, 4)$

$t = \pi \to (-6, 1)$



c) $A = \int_0^{\pi} 2\pi\,(1 + 3\sin t)\sqrt{16\sin^2 t + 9\cos^2 t}\, dt$

Q2.

a) Identify the symmetries of the curve $r^2 = -cos\theta$.

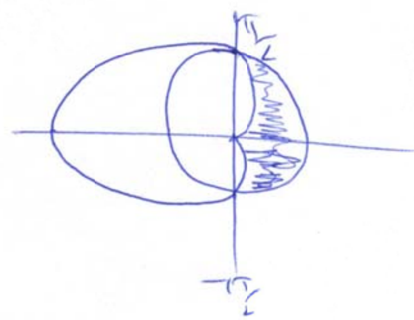
b) Find the area of the region that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - cos\theta$.

a) x-axis $(r, -\theta) \rightarrow r^2 = -\cos(-\theta) \rightarrow r^2 = -\cos\theta$ ✓

y-axis $(-r, -\theta) \rightarrow (-r)^2 = -\cos(-\theta) \rightarrow r^2 = -\cos\theta$ ✓

origin

b) Intersections



$1 = 1 - \cos\theta \rightarrow \cos\theta = 0$

$\theta = \frac{\pi}{2}, -\frac{\pi}{2}$

$$A = 2 \int_0^{\frac{\pi}{2}} \frac{1}{2}\left[1^2 - (1 - \cos\theta)^2\right] d\theta$$

$$= \int_0^{\pi/2} (1 - 1 + 2\cos\theta - \cos^2\theta)\, d\theta$$

$$= 2\sin\theta \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \frac{1 + \cos 2\theta}{2}\, d\theta$$

$$= 2 - \frac{1}{2}\left[\theta + \frac{\sin 2\theta}{2} \Big|_0^{\frac{\pi}{2}}\right]$$

$$= 2 - \frac{\pi}{4}$$

[redacted]