King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

MATH 201 - Exam II - Term 141

Duration: 120 minutes

Name: Solution

ID Number:

Section Number:

Serial Number:

Class Time:

Instructor's Name:

Instructions:

1. Calculators and Mobiles are not allowed.
2. Write neatly and eligibly. You may lose points for messy work.
3. Show all your work. No points for answers without justification.
4. Make sure that you have 10 pages of problems, **10 MCQS and 5 Written Questions** (Total of 15 Problems)

Page Number	**Points**	**Maximum Points**
1		10
2		10
3		10
4		10
5		10
6		10
7		14
8		6
9		10
10		10
Total		100

1. If the plane $ax + 3y - 2z = b$ contains the line $L : x = 3 - t, y = 2t, z = t$, then $a + b$ is equal to

(a) 16

(b) 12

(c) 4

(d) 6

(e) 8

2. The surface $x^2 - 4y^2 + z^2 - 6x - 8y - 2z + 6 = 0$ is:

(a) A circular cone

(b) An elliptical paraboloid

(c) An ellipsoid

(d) A Hyperbolic paraboloid

(e) An elliptical cone

3. For the function $f(x,y) = \dfrac{x^2 - y^2}{1 + xy}$, we have that

(a) $\lim_{(x,y)\to(1,-1)} f(x,y)$ does not exist

(b) $\lim_{(x,y)\to(1,-1)} f(x,y) = 2$

(c) $\lim_{(x,y)\to(1,-1)} f(x,y) = -2$

(d) $\lim_{(x,y)\to(1,-1)} f(x,y) = 0$

(e) $\lim_{(x,y)\to(1,-1)} f(x,y) = -1$

4. The range of $f(x,y,z) = \dfrac{1}{\ln\sqrt{9 - x^2 - y^2 - z^2}}$ is

(a) $(-\infty, 0) \cup \left[\frac{1}{\ln 3}, \infty\right)$

(b) $\left(0, \frac{1}{\ln 3}\right]$

(c) $(-\infty, 0) \cup \left(0, \frac{1}{\ln 3}\right)$

(d) $(-\infty, \infty)$

(e) $\left(-\infty, \frac{1}{\ln 3}\right]$

5. Let $f(x,y) = \int_x^y \cos t\, dt$. Then $f_x\left(\frac{\pi}{3}, 0\right) - f_y\left(0, \frac{\pi}{3}\right)$ is equal to

(a) -1

(b) -2

(c) $\frac{1}{2}$

(d) 1

(e) $\sqrt{3}$

6. For $w = x^2 + yz$ with $x = 3t^2 + 1, y = 2t - 4, z = t^3$, the value of $\frac{dw}{dt}$ at $t = 1$ is:

(a) 44

(b) 0

(c) -12

(d) 1

(e) $\frac{2}{5}$

7. The directions of zero change in $f(s,t) = \frac{1}{2}(s^2+t^2)$ at $(1,1)$ are

(a) $\left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$ and $\left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$

(b) $\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$ and $\left\langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$

(c) $\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$ and $\langle 0, 1 \rangle$

(d) $\left\langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$ and $\langle 1, 0 \rangle$

(e) $\langle 0, -1 \rangle$ and $\langle -1, 0 \rangle$

8. The plane tangent to the surface $z = x\cos y - ye^x$ at $(0,0,0)$ is

(a) $x - y - z = 0$

(b) $x + y + z = 0$

(c) $-2x + y - z = 0$

(d) $x + y - z = 0$

(e) $x - y + 2z = 0$

9. Let $w = x + y$ where $x = ln\sec^2\frac{t}{2}$ and $y = \sin t$.
If $\frac{dw}{dt}|_{t=\alpha} = 1$, then $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is equal to:

(a) 0

(b) -1

(c) $\frac{1}{2}$

(d) 1

(e) $-\frac{1}{2}$

10. If $xe^y + ye^z + 2(\ln x - 1) = 0$, then the value of $\frac{\partial z}{\partial x}|_{(1,\ln 2,\ln 3)}$ is

(a) $\frac{-4}{\ln 8}$

(b) $\frac{-4}{\ln 4}$

(c) $\frac{-8}{\ln 8}$

(d) $\frac{4}{\ln 8}$

(e) $\frac{5}{3\ln 2}$

11. **(10 points)** Find an equation of a plane through points $A(1,-2,0)$ and $B(2,0,3)$ and parallel to the line $L: x=-3+2t, y=3t+1, z=4t$.

Sol: $\vec{AB} = \langle 2-1, 0+2, 3-0 \rangle = \langle 1, 2, 3 \rangle$ lies in the plane. (2)

The vector $\vec{d} = \langle 2, 3, 4 \rangle$ is parallel to the line L. (1)

Hence $\vec{AB} \times \vec{d}$ is normal to the plane. (2)

$$\vec{AB} \times \vec{d} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix} = (8-9)\vec{i} - (4-6)\vec{j} + (3-4)\vec{k}$$

$$= \langle -1, 2, -1 \rangle \quad (3)$$

Using the point $A(1,-2,0)$, we find the plane equation

$$-1(x-1) + 2(y+2) - 1(z-0) = 0 \quad (2)$$

$$\Rightarrow \quad x - 2y + z = 5$$

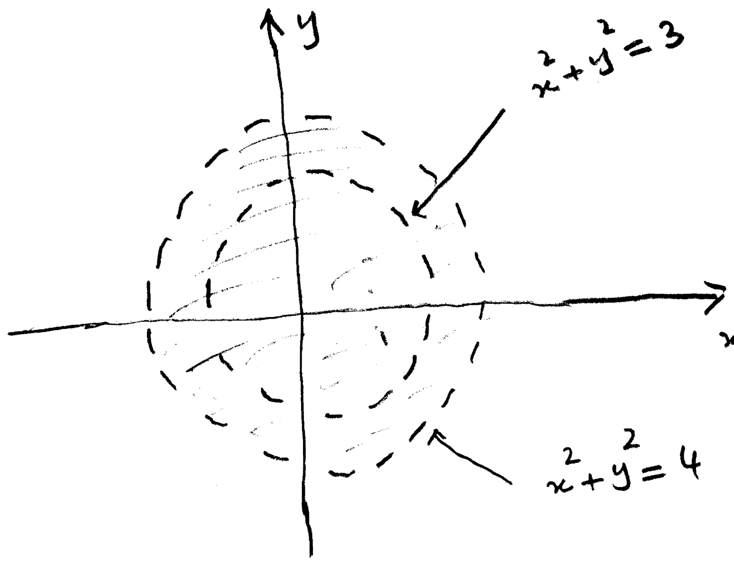
12. Consider the function

$$f(x,y) = \frac{1}{\ln\sqrt{4-x^2-y^2}}$$

a) **(7 points)** Find and sketch the domain of f.

b) **(7 points)** Find and sketch the level curve of f that passes through $(0,1)$.

Sol: a) The domain is $D = \{(x,y): x^2+y^2 < 4,\ x^2+y^2 \neq 3\}$ (2) (2)



(3)

b) As the level curve $f(x,y)=c$ passes through $(0,1)$, so

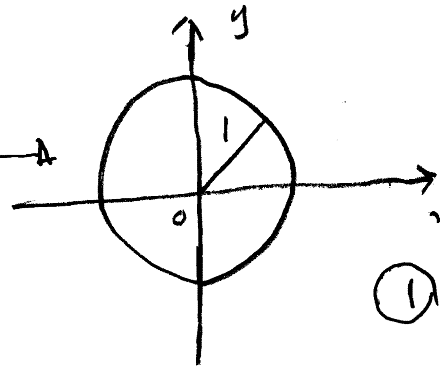
$$f(0,1) = c \Rightarrow c = \frac{1}{\ln\sqrt{4-0-1}} = \frac{1}{\ln\sqrt{3}}$$ (3)

The level curve is

$$\frac{1}{\ln\sqrt{4-x^2-y^2}} = \frac{1}{\ln\sqrt{3}} \Rightarrow 4-x^2-y^2 = 3$$ (3)

$$\Rightarrow x^2+y^2 = 1$$

Sketch of level curve



(1)

13. **(6 points)** Let $f(x,y) = \sin\left(\frac{x}{y}\right) + e^{x/y} + 5x$. Show that $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 5x$.

Sol:

$$\frac{\partial f}{\partial x} = \frac{1}{y}\cos\left(\frac{x}{y}\right) + \frac{1}{y}e^{\frac{x}{y}} + 5$$ (2)

$$\frac{\partial f}{\partial y} = -\frac{x}{y^2}\cos\left(\frac{x}{y}\right) - \frac{x}{y^2}e^{\frac{x}{y}}$$ (2)

Therefore

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = \frac{x}{y}\cos\left(\frac{x}{y}\right) + \frac{x}{y}e^{\frac{x}{y}} + 5x - \frac{x}{y}\cos\left(\frac{x}{y}\right) - \frac{x}{y}e^{\frac{x}{y}}$$

$$= 5x$$ (2)

14. **(10 points)** By about how much will $f(x,y,z) = \ln\sqrt{x^2+y^2+z^2}$ change if the point $P(x,y,z)$ moves from $P_0(3,4,12)$ a distance of $ds = 0.1$ unit in the direction of $3\vec{i}+6\vec{j}-2\vec{k}$?

Sol : $\left|3\vec{i}+6\vec{j}-2\vec{k}\right| = \sqrt{9+36+4} = \sqrt{49} = 7$

Direction of motion of $p(x,y,z)$ is

$\vec{u} = \left\langle \frac{3}{7}, \frac{6}{7}, -\frac{2}{7} \right\rangle$ (2)

(2)

$\Delta f \approx df = \left(\nabla f|_{P_0} \cdot \vec{u}\right) ds \longrightarrow (*)$

Now $\left.\frac{\partial f}{\partial x}\right|_{P_0} = \left.\frac{x}{x^2+y^2+z^2}\right|_{P_0} = \frac{3}{9+16+144} = \frac{3}{169}$ (1)

$\left.\frac{\partial f}{\partial y}\right|_{P_0} = \left.\frac{y}{x^2+y^2+z^2}\right|_{P_0} = \frac{4}{169}$ (1)

$\left.\frac{\partial f}{\partial y}\right|_{P_0} = \left.\frac{z}{x^2+y^2+z^2}\right|_{P_0} = \frac{12}{169}$ (1)

(1)

$\Rightarrow \nabla f|_{P_0} = \left\langle \frac{3}{169}, \frac{4}{169}, \frac{12}{169} \right\rangle$

From (*), we get

$df = \frac{1}{7\times 169}\left[\langle 3,4,12\rangle \cdot \langle 3,6,-2\rangle\right](0.1)$

$= \frac{1}{1183}(9+24-24)\times\frac{1}{10} = \frac{9}{11830}$ (2)

15. **(10 points)** Show that the curve $\vec{r}(t) = \sqrt{t}\,\vec{i} + \sqrt{t}\,\vec{j} + (2t-1)\,\vec{k}$ is tangent to the surface $x^2 + y^2 - z = 1$ when $t = 4$.

<u>Sol</u>: $\vec{r}(t)$ is tangent to $f(x,y,z)=c$ at $t=t_0 \iff \nabla f\big|_{t=t_0} \cdot \vec{V}(t_0) = 0$

$$\vec{r}(4) = 2\vec{i} + 2\vec{j} - 7\vec{k} \Rightarrow P_0 = (2,2,-7) \quad ②$$

$$\vec{V}(t) = \frac{1}{2\sqrt{t}}\vec{i} + \frac{1}{2\sqrt{t}}\vec{j} + 2\vec{k} \quad ②$$

$$\Rightarrow \vec{V}(4) = \frac{1}{4}\vec{i} + \frac{1}{4}\vec{j} + 2\vec{k} \quad ①$$

Here $f(x,y,z) = x^2 + y^2 - z - 1 = 0$

$$\Rightarrow \nabla f(x,y,z) = 2x\vec{i} + 2y\vec{j} - \vec{k}$$

$$\Rightarrow \nabla f(2,2,-7) = 4\vec{i} + 4\vec{j} - \vec{k} \quad ②$$

Now $\nabla f(2,2,-7) \cdot \vec{V}(4) = (4\vec{i} + 4\vec{j} - \vec{k}) \cdot (\frac{1}{4}\vec{i} + \frac{1}{4}\vec{j} + 2\vec{k})$

$$= 1 + 1 - 2$$

$$= 0$$

Hence $\vec{r}(t)$ is tangent to the surface

$x^2 + y^2 - z = 1$ at $t = 4$. ③

Q	MM	V1	V2	V3	V4
1	a	a	c	e	a
2	a	d	a	a	d
3	a	b	d	c	b
4	a	b	d	c	c
5	a	d	e	b	e
6	a	a	d	c	c
7	a	d	b	d	c
8	a	b	e	d	b
9	a	d	e	a	c
10	a	e	d	d	a