KFUPM MATH201 Quiz#3 Time:20min. Sec.11

Q1 Find the equation of the line in which the planes intersect.

$5x - 2y = 11, \quad 4y - 5z = -17$

60. $\mathbf{n}_1 = 5\mathbf{i} - 2\mathbf{j}$ and $\mathbf{n}_2 = 4\mathbf{j} - 5\mathbf{k} \Rightarrow \mathbf{n}_1 \times \mathbf{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & -2 & 0 \\ 0 & 4 & -5 \end{vmatrix} = 10\mathbf{i} + 25\mathbf{j} + 20\mathbf{k}$, the direction of the desired line; $(1, -3, 1)$ is on both planes $\Rightarrow$ the desired line is $x = 1 + 10t$, $y = -3 + 25t$, $z = 1 + 20t$

intersection point let $x = 1 \rightarrow y = -3 \rightarrow z = 1 \rightarrow (1, -3, 1)$

Q2. Find the limit $\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^2+y^2}$ if it exists.

Change to polar coordinates $\lim_{r\to 0} \frac{r^3 cos^2\theta sin\theta}{r^2} = \lim_{r\to 0} r cos^2\theta sin\theta = 0$

$cos^2\theta sin\theta$ bouded

Q3. $z = \tan^{-1}\left(\frac{x}{y}\right), \quad x = u\cos(v), \ y = u\sin(v)$. Express $\frac{\partial z}{\partial u}$ in terms of u and v

$$8.\ (a)\ \frac{\partial z}{\partial u} = \left[\frac{\left(\frac{1}{y}\right)}{\left(\frac{x}{y}\right)^2 + 1}\right]\cos v + \left[\frac{\left(\frac{-x}{y^2}\right)}{\left(\frac{x}{y}\right)^2 + 1}\right]\sin v = \frac{y\cos v}{x^2+y^2} - \frac{x\sin v}{x^2+y^2} = \frac{(u\sin v)(\cos v) - (u\cos v)(\sin v)}{u^2} = 0;$$