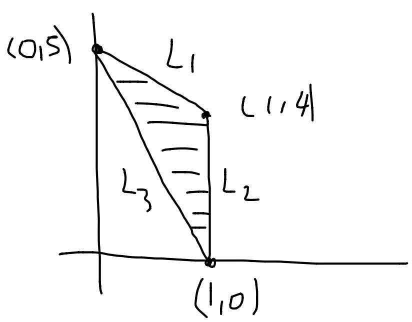
KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
MATH 201 - QUIZ 4

Name:
Student ID #:

Question 1. Find the absolute maximum and minimum values of $f(x, y) = 3 + xy - x - 2y$ on the closed triangle with vertices $(1, 0)$, $(0, 5)$, and $(1, 4)$.
Question 2. Find the point closest to the origin on the plane $2x + y - 3z = 4$.

Your Solution.

<u>Question 1</u>



We need to find the equation of the edges.

$\underline{L_1}$: slope of $L_1 = \frac{5-4}{0-1} = -1$ $y - 5 = -1(x-0) \Rightarrow y = 5 - x$.

$\underline{L_2}$: $x = 1$

$\underline{L_3}$: slope of L_3: $\frac{5-0}{0-1} = -5$ $y - 5 = -5(x-0) \Rightarrow y = 5 - 5x$

<u>inside the triangle</u> !

$f_x(x,y) = y - 1 = 0 \Rightarrow y = 1$
$f_y(x,y) = x - 2 = 0 \Rightarrow x = 2$) the point $(2,1)$ is not inside the triangle, therefore

f has no CP.

<u>on the boundary</u>.

<u>on L_1</u>; $g(x) = f(x, 5-x) = 3 + x\cdot(5-x) - x - 2(5-x) = 3 + 5x - x^2 - x - 10 + 2x = -x^2 + 6x - 7$

on the interval $[0,1]$.

$g'(x) = -2x + 6 = 0 \Rightarrow x = 3 \rightsquigarrow$ not in the interval $\rightsquigarrow$ NO CP.

$g(0) = -7$ $g(1) = -2$.

<u>on L_2</u>: $h(y) = f(1,y) = 3 + y - 1 - 2y = 2 - y$ on the interval $[0,4]$.

$h'(y) = -1 \rightsquigarrow$ NO C.P.

$h(0) = 2$ $h(4) = -2$.

on L_3 $k(x) = f(x, 5-5x) = 3 + x(5-5x) - x - 2(5-5x) = 3+5x-5x^2-x-10+10x$

$= -5x^2 + 14x - 7$ on the interval $[0,1]$.

$k'(x) = -10x + 14 = 0 \quad x = \frac{14}{10}$ not in the interval $\rightarrow$ NO CP.

$k(0) = -7 \quad k(1) = -5+14-7 = 2.$

So ahs max value is 2 and ahs minimum value is -7.

Question 2

This question wants us to find the point (x,y,z) on the plane $2x+y-3z=4$ that minimizes the distance function $f(x,y,z) = x^2+y^2+z^2$. The constraint func in this case is $g(x,y) = 2x+y-3z-4$

$\nabla f = \langle 2x, 2y, 2z \rangle \quad \nabla g = \langle 2, 1, -3 \rangle.$

We need to solve the system $\begin{cases} 2x = 2\lambda \\ 2y = \lambda \\ 2z = -3\lambda \\ 2x+y-3z=4 \end{cases} \Rightarrow \begin{array}{l} x = \lambda \\ y = \lambda/2 \\ z = -3\lambda/2 \end{array}$

$2\lambda + \frac{\lambda}{2} + \frac{9\lambda}{2} = 4 \Rightarrow 7\lambda = 4$

$\lambda = 4/7.$

Then $x = 4/7 \quad y = 2/7 \quad z = -6/7$