Chapter 9

$$\bar{x}_{\alpha L} = \mu - z_\alpha \frac{\sigma}{\sqrt{n}}, \quad \bar{x}_{\alpha U} = \mu + z_\alpha \frac{\sigma}{\sqrt{n}}$$

$$\bar{x}_{\frac{\alpha}{2}} = \mu \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

$$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \text{ or } t_{n-1} = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$$

$$z = \frac{p-\pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} \text{ where } p = \frac{x}{n}$$

Chapter 10

The i[th] **paired difference** $d_i = x_{1i} - x_{2i}$.

Test statistic is $t_{n-1} = \frac{\bar{d}-\mu_d}{s_d/\sqrt{n}}$ $\quad \bar{d} = \frac{\sum_{i=1}^{n} d_i}{n}$ and

$$s_d = \sqrt{\frac{\sum_{i=1}^{n}\left(d_i - \bar{d}\right)^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{n} d_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} d_i\right)^2}{n-1}}.$$

$$z = \frac{\left(\bar{x}_1 - \bar{x}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$z = \frac{\left(\bar{x}_1 - \bar{x}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$t_{n_1+n_2-2} = \frac{\left(\bar{x}_1 - \bar{x}_2\right) - \left(\mu_1 - \mu_2\right)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where $s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$

$$t_{df} = \frac{\left(\bar{x}_1 - \bar{x}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

where $df = \frac{\left(s_1^2/n_1 + s_2^2/n_2\right)^2}{\left(\frac{\left(s_1^2/n_1\right)^2}{n_1-1} + \frac{\left(s_2^2/n_2\right)^2}{n_2-1}\right)}$

$$\bar{p} = \frac{n_1 p_1 + n_2 p_2}{n_1+n_2} = \frac{x_1+x_2}{n_1+n_2}$$

$$z = \frac{(p_1-p_2)-(\pi_1-\pi_2)}{\sqrt{\bar{p}\left(1-\bar{p}\right)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$F_0 = \frac{s_1^2}{s_2^2} \qquad df_1 = n_1 - 1 \;\; df_2 = n_2 - 1.$$

Chapter 12

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

$$\chi^2 = \sum_{j=1}^{c} \frac{\left(o_j - e_j\right)^2}{e_j} > \chi_\alpha^2$$

$$|p_i - p_j| > \sqrt{\chi_\alpha^2}\sqrt{\frac{p_i(1-p_i)}{n_i} + \frac{p_j(1-p_j)}{n_j}}$$

$$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{c} \frac{\left(o_{ij} - e_{ij}\right)^2}{e_{ij}}$$

$$e_{ij} = \frac{(i^{th}\text{ Row total})(j^{th}\text{ Column total})}{\text{Total sample size}}$$

$$z = \frac{B-C}{\sqrt{B+C}}$$

$$\chi^2 = \sum_{i=1}^{k} \frac{\left(o_i - e_i\right)^2}{e_i}$$

Chapter 13

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2 \sum(y-\bar{y})^2}}$$

$$= \frac{\sum xy - \left(\sum x\right)\left(\sum y\right)/n}{\sqrt{\left[\sum x^2 - \left(\sum x\right)^2/n\right]\left[\sum y^2 - \left(\sum y\right)^2/n\right]}} = \frac{Sxy}{\sqrt{Sxx\,Syy}}$$

$$t_{n-2} = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}}$$

$$\hat{y}_i = b_0 + b_1 x$$

$$b_1 = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$$

$$b_1 = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{\left(\sum x\right)^2}{n}} = \frac{Sxy}{Sxx}$$

And $b_0 = \bar{y} - b_1\bar{x}$

Error $e_i = Y_i - Y_i$

$$SST = Syy = \sum(y-\bar{y})^2$$

$$= \sum_1^n y_i^2 - n\bar{y}^2$$

$$SSR = \sum(\hat{y}-\bar{y})^2$$

Sum of Squares Error $SSE = SST - SSR$

$$SSE = Syy - \frac{(Sxy)^2}{Sxx}$$

(ALSO $SSE = \sum(y-\hat{y})^2$)

$$Sxy = \sum xy - (\sum x)(\sum y)/n$$

$$R^2 = \frac{SSR}{SST}$$

$$R^2 = \frac{SST - SSE}{SST} = 1 - \frac{SSE}{SST} = r^2$$

$$s_\varepsilon = \sqrt{\frac{SSE}{n-1-1}}$$

$$s_{b_1} = \frac{s_\varepsilon}{\sqrt{\sum(x-\bar{x})^2}} = \frac{s_\varepsilon}{\sqrt{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}} = \frac{s_\varepsilon}{\sqrt{Sxx}}$$

$$t_{n-2} = \frac{b_1 - \beta_1}{s_{b_1}} \quad \& \quad b_1 \pm t_{\alpha/2} s_{b_1}$$

$$\hat{y} \pm t_{\alpha/2} s_\varepsilon \sqrt{\frac{1}{n} + \frac{(x_p-\bar{x})^2}{\sum(x-\bar{x})^2}}$$

$$\hat{y} \pm t_{\alpha/2} s_\varepsilon \sqrt{1 + \frac{1}{n} + \frac{(x_p-\bar{x})^2}{\sum(x-\bar{x})^2}}$$

Chapter 13, 14, 15 & 16

$$d = \frac{\sum_{t=1}^{n}(e_t - e_{t-1})^2}{\sum_{t=1}^{n} e_t^2},$$

Chapter 14 & 15

$$\hat{y} = b_0 + b_1x_1 + b_2x_2 + \ldots + b_kx_k$$

$$\hat{y} = b_0 + b_1x_1 + b_2x_2 \quad \& \quad e_i = y_i - \hat{y}_i$$

$$R_A^2 = 1 - (1-R^2)\left(\frac{n-1}{n-k-1}\right)$$

Test statistic $F = \frac{\frac{SSR}{k}}{\frac{SSE}{n-k-1}} = \frac{MSR}{MSE}$

$$SSR(X_j \mid \text{All except } X_j) = SSR(\text{All}) - SSR(\text{All except } X_j)$$

$$r^2_{YX_j \bullet (\text{All except } X_j)}$$

$$= \frac{SSR(X_j \mid \text{All except } X_j)}{SST - SSR(\text{All}) + SSR(X_j \mid \text{All except } X_j)}$$

$$t_{n-k-1} = \frac{b_i - 0}{s_{b_i}} \quad \text{or} \quad F_{1,n-k-1} = \frac{SSR(X_j \mid \text{All except } X_j)}{MSE}$$

$$t^2_{n-k-1} = F_{1,n-k-1}$$

$$b_i \pm t_{\alpha/2} s_{b_i},.$$

$$s_\varepsilon = \sqrt{\frac{SSE}{n-k-1}} = \sqrt{MSE}$$

$$VIF_j = \frac{1}{1-R_j^2}$$

$$SSR(X_j \mid \text{All except } X_j) = SSR(\text{All}) - SSR(\text{All except } X_j)$$

$$C_p = \frac{(1-R_k^2)(n-T)}{1-R_T^2} - [n-2(k+1)]$$

$$F = \frac{\frac{SSR_{Full} - SSR_{Reduced}}{m}}{\frac{SSE}{n-k-1}} = \frac{\frac{SSE_{Reduced} - SSE_{Full}}{m}}{MSE}$$

with $df_1 = m$ and $df_2 = n-k-1$

Chapter 16

$$I_t = \frac{y_t}{y_0}100 \quad \& \quad I_t = \frac{\sum p_t}{\sum p_0}100$$

$$I_t = \frac{\sum q_t p_t}{\sum q_t p_0}100 \quad \& \quad I_t = \frac{\sum q_0 p_t}{\sum q_0 p_0}100$$

$$y_{adj_t} = \frac{y_t}{I_t}100$$

$$F_t = \hat{y} = b_0 + b_1 t \quad \& \quad e_t = y_t - F_t$$

$$MSE = \frac{\sum (y_t - F_t)^2}{n} \quad \& \quad MAD = \frac{\sum |y_t - F_t|}{n}$$

$$y_t = T_t \times S_t \times C_t \times I_t$$

$$S_t \times I_t = \frac{y_t}{T_t \times C_t} \quad \& \quad T_t \times C_t \times I_t = \frac{y_t}{S_t}$$

$$F_{t+1} = F_t + \alpha(y_t - F_t) = \alpha y_t + (1-\alpha)F_t$$

$$F_{t+1} = C_t + T_t \quad \text{where } C_t = \alpha y_t + (1-\alpha)(C_{t-1} + T_{t-1})$$

$$\& \quad T_t = \beta(C_t - C_{t-1}) + (1-\beta)T_{t-1}$$

$$Y_t = \beta_o \beta_1^{X_t} \varepsilon_t$$

$$\log(Y_t) = \log(\beta_o) + X_t \log(\beta_1) + \log(\varepsilon_t)$$

$$Y_t = \beta_o \beta_1^{X_t} \beta_2^{Q_1} \beta_3^{Q_2} \beta_4^{Q_3} \varepsilon_t$$

$$\log(Y_t) = \log(\beta_o) + X_t \log(\beta_1) + Q_1 \log(\beta_2)$$
$$+ Q_2 \log(\beta_3) + Q_3 \log(\beta_4) + \log(\varepsilon_t)$$

$$Y_t = A_o + A_1 Y_{t-1} + A_2 Y_{t-2} \dots + A_p Y_{t-p} + \varepsilon_t$$

Some Useful STAT211 Formulas

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} \quad \text{or} \quad \frac{\sum_{i=1}^{n} x_i^2 - \frac{1}{n}(\sum_{i=1}^{n} x_i)^2}{n-1}$$

Binomial $P(x) = \frac{n!}{x!(n-x)!} p^x q^{n-x}$,

$\mu_X = E[X] = np, \ \sigma_X = \sqrt{npq}$

Poisson $P(x) = \frac{(\lambda t)^x}{x!} e^{-\lambda t}$,

$\mu_X = E[X] = \lambda t, \ \sigma_X = \sqrt{\lambda t}$

Hypergeometric $P(x) = \frac{C_{n-x}^{N-X} C_x^X}{C_n^N}$,

Discrete Uniform $P(x) = \frac{1}{k}$, for k discrete points

<u>Continuous</u>

Uniform function $f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \le x \le b \\ 0 & \text{otherwise} \end{cases}$

$\mu_X = E[X] = \frac{a+b}{2}, \ \sigma_X = \sqrt{\frac{(b-a)^2}{12}}$

Exponential function $f(x) = \lambda e^{-\lambda x}$,

$P(0 \le x \le a) = 1 - e^{-\lambda a}, \ \mu_X = E[X] = \frac{1}{\lambda}, \ \sigma_X = \frac{1}{\lambda}$