

Final Version

**King Fahd University of Petroleum & Minerals**  
**Department of Mathematics & Statistics**  
**Math 302 Final Exam**  
**The Third Semester of 2013-2014 (133)**

**Time Allowed: 150 Minutes**

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Name: \_\_\_\_\_ ID#: \_\_\_\_\_

Section/Instructor: \_\_\_\_\_ Serial #: \_\_\_\_\_

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- Mobiles and calculators are not allowed in this exam.
  - Write neatly and eligibly. You may lose points for messy work.
  - Show all your work. No points for answers without justification.
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Question #	Marks	Maximum Marks
1		15
2		15
3		20
4		15
5		15
6		15
7		15
8		15
9		15
Total		140

**Q:1** (15 points) Determine whether the set  $S$  of vectors of the form  $(x, x + y, 2y, 0)$  (where  $x, y$  are real numbers) is a subspace of  $\mathbb{R}^4$ . If yes, what is its dimension? find a basis for  $S$ . Is the basis unique? if no, find a second basis.

**Q:2** (15 points) Find the rank of the matrix  $A = \begin{pmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{pmatrix}$

**Q:3 a** (8 points) Find the values of  $(1 - i)^{2i}$ .

**Q:3 b** (12 points) Solve  $\cos z = -3$ .

**Q:4** (15 points) Show that  $u(x, y) = \cos x \cosh y$  is a harmonic function and determine  $v(x, y)$  so that  $w = u + iv$  is an analytic function.

**Q:5** (15 points) Find all Laurent's series expansion of the function  $f$  around  $z = 0$  and specify in which annuli they converge;  $f$  being defined by

$$f(z) = \frac{1}{z^2(z-1)}$$

**Q:6** (15 points) Show that  $z = 0$  is removable singularity of  $f(z) = \frac{\sin 4z - 4z}{z^2}$ .

**Q:7** (15 points) Evaluate

$$\oint_{|z-1|=3} \frac{e^{2z}}{(z-2)^2} dz$$



**Q:8** (15 points) Evaluate

$$\oint_{|z|=3} \frac{e^z}{(z-1)(z-2)} dz$$

**Q:9** (15 points) Evaluate the Cauchy principal value of

$$\int_{-\infty}^{\infty} \frac{1}{(x^2 + 1)^3} dx.$$