Final Version

King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 302 Major Exam II
The Third Semester of 2013-2014 (133)

Time Allowed: 90 Minutes

Name: ______________________ ID#: ______________

Section/Instructor: ______________ Serial #: ______________

- Mobiles and calculators are not allowed in this exam.
- Write neatly and eligibly. You may lose points for messy work.
- Show all your work. No points for answers without justification.

Question #	Marks	Maximum Marks
1		20
2		20
3		10
4		16
5		16
6		18
Total		100

Q:1 (20 points) Let D be the region bounded by the hemisphere $x^2 + y^2 + (z-1)^2 = 9$, $1 \leq z \leq 4$. Verify the divergence theorem if $\mathbf{F} =< x, y, z-1 >$.

Q:2 (20 points) (a) Find the basis and dimension of subspace
$S = \{< x, y, z, w > \mid x + y + z = w \text{ and } x, y, z, w \in R\}$ of R^4.

(b) Let S $= \{(x, y) \in R^2 \mid x = 2y + 1\}$. Determine whether S is a subspace of R^2.

Q:3 (10 points) Let A be a non-zero 4×6 matrix .
(a) What is the maximum rank that A can have?

(b) If $\text{rank}(A|B) = 2$, then for what value(s) of rank A is the system $AX = B,\ B \neq 0$ inconsistent? consistent?

(C) If rank $(A) = 3$, then how many parameters does the solution of the system $AX = 0$ have?

Q:4 (16 points) Use Gauss-Jordan elimination to find inverse of $A = \begin{pmatrix} 2 & 0 & 1 \\ -2 & 3 & 4 \\ -5 & 5 & 6 \end{pmatrix}$

Q:5 (16 points) Let $A = \begin{pmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{pmatrix}$

(a) Verify that eigenvalues of A are $\lambda_1 = 0, \lambda_2 = -4$ and $\lambda_3 = 3$.

(b)Find an eigenvector corresponding to λ_2.

Q:6 (18 points) Let $A = \begin{pmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{pmatrix}$

(a) Explain briefly why A is diagonalizable.

(b) If eigenvalues and corresponding eigenvectors of A are

$$\lambda_1 = 11,\ \lambda_2 = \lambda_3 = 8,\ K_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix},\ K_2 = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix},\ K_3 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix},$$

then find an orthogonal matrix P that diagonalizes A, and find the matrix $P^{-1}AP$.