

King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 302 Major Exam I
The Third Semester of 2013-2014 (133)

Time Allowed: 120 Minutes

Name: _____ ID#: _____

Section/Instructor: _____ Serial #: _____

- Mobiles and calculators are not allowed in this exam.
 - Write all steps clear.
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Question #	Marks	Maximum Marks
1		8
2		6
3		14
4		16
5		12
6		14
7		16
8		14
Total		100

Q:1 (8 points) Let C be a curve traced by the vector function $\mathbf{r}(t) = \langle 3t, \sqrt{3} t^2, \frac{2}{3}t^3 \rangle$, $0 \leq t \leq 1$. Evaluate the length of the curve C .

Q:2 (6 points) Find the maximum rate of change of $f(x, y) = \frac{y^2}{x}$ at the point $(2, 4)$.

Q:3 (14 points) Find the points on the hyperboloid $x^2 - y^2 + 2z^2 = 1$ where the normal line is parallel to the line that joins the points $(3, -1, 0)$ and $(5, 3, 6)$.

Q:4 (16 points) Determine if the vector field

$$\mathbf{F}(x, y) = \langle 2x - y \sin(xy) - 5y^4, -20xy^3 - x \sin(xy) \rangle$$

is conservative field. If so, find a potential function $\phi(x, y)$ for \mathbf{F} and evaluate the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is any path from $(-2, 0)$ to $(1, 0)$.

Q:5(a) (8 points) Use Green Theorem to evaluate

$$\oint_C (3y - e^{\sin x})dx + (7x + \sqrt{y^2 - 1})dy$$

where C is the circle $x^2 + y^2 = 9$ oriented positively.

Q:5(b) (4 points) If $\phi = 2x^3y^2z^4$, find $\text{div}(\text{grad } \phi)$.

Q:6 (14 points) Evaluate the surface integral $\iint_S yz \, dS$, where S that portion of the cone $z = \sqrt{x^2 + y^2}$ lies between the plane $z = \frac{1}{2}$ and $z = 1$.

Q:7 (16 points) Verify Green Theorem when the vector field is $\mathbf{F} = \langle x, y \rangle$, and the curve C consists of the line segment from $(0, 1)$ to $(0, 0)$ and from $(0, 0)$ to $(1, 0)$ and the parabola $y = 1 - x^2$ from $(1, 0)$ to $(0, 1)$.

Q:8 (14 points) Use Stokes' Theorem to compute $\int \int_S \text{curl} \mathbf{F} \cdot \mathbf{n} \, dS$, where $\mathbf{F}(x, y, z) = \langle xz, yz, xy \rangle$ and S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy - plane.