KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

Department of Mathematics and Statistics

MATH 260-(133)

Major Exam 2

Code 001

Time: 120 Minutes **Maximum Points: 100**

Name: I.D. # Section: Ser.#

Show All Necessary Work

Calculators are not allowed in this exam

PART-I (MCQ) Each MCQ = 6 points]

Encircle your choice (answer of each MCQ) in the following Table

QCQ #	Answer				
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E

PART-II (Written)

Q	Points
1	/12
2	/10
3	/12
4	/12
5	/12
6	/12
Total	/70

PART I, MCQ: Encircle your answer in the Table on the Cover Page.

Q1. Let $A=(a_{ij}), a_{ij}=i+j$ and $B=(b_{jk}), b_{jk}=j-k$ be 8×8 matrices. If $C=AB=(c_{ik})$, then the coefficient c_{33} is equal to: (Hint $\sum_{j=1}^{j=n} j^2 = \frac{n(n+1)(2n+1)}{6}$)

A) 132.
B) 195.
C) 204.
D) -132.
E) -195.

Q2. For which value(s) of k, the vectors $v_1=(1,1,k,1), v_2=(1,k,1,0), v_3=(1,1,k,0)$ and $v_4=(1,0,0,0)$ form a basis for the vector space R^4.

A) all real numbers.
B) all real numbers except 1.
C) all real numbers except 1 and -1
D) all real numbers except -1
E) all real numbers except 0.

Q3. For which value(s) of k, the set $W=\{(x,y,z)\,|\,x+y+z=k^2-4\}$ is a subspace of R^3.

A) all real numbers.
B) $k=2$.
C) $k=-2$
D) $k=\pm 2$
E) all real numbers except -2 and 2.

Q4. If $y_1 = x$ and $y_2 = x\ln x$ are solutions of the differential equation $x^2y''-xy'+y=0$, then the initial value problem $x^2y''-xy'+y=0$, $y'(e)=4, y(e)=e$ has a solution of the form:

$A)$ $y = 2x + 3x\ln x$

$B)$ $y = -2x + 3x\ln x$

$C)$ $y = 2x - 3x\ln x$

$D)$ $y = -2x - 3x\ln x$

$E)$ $y = x + 3x\ln x$

Q5. If A and B are two 3×3 matrices with $\det A = 2$ and $\det B = 4$, then $\det(2AB)+\det(A^{-1}B^T)$ is:

$A)$ 64

$B)$ 18

$C)$ 72

$D)$ $\frac{513}{8}$

$E)$ 66

Part II. Provide <u>complete</u> solution of each question <u>showing all necessary steps.</u>

Q 1. a/ Use the method of cofactors to find the inverse A^{-1} of the matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \\ 1 & 1 & 2 \end{pmatrix}$

b/ Find a matrix X such that $AX = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 4 & -1 \\ 5 & 0 & -2 \end{pmatrix}$

Q 2. Use Cramer rule to solve the system (S): $\begin{cases} 2x + 3y - 3z = 1 \\ x + 2y + 2z = 0 \\ x - y + z = 0 \end{cases}$

Q 3. a/ Solve the differential equation: $y'''-5y''+8y'-4y=0$.

b/ Find a homogeneous differential equation with constant coefficients whose solution is $y(x)=c_0e^x+c_1xe^x+c_2e^{2x}$.

Q 4. Use the method of undetermined coefficient to solve the differential equation: $y''-4y'-5y = xe^{x} + \sin x$
[Do not Evaluate the Constants for the particular solution]

Q 5. a/ Find a basis and the dimension of the solution space of the system

$$\begin{cases} x + y + z + t = 0 \\ 2x + y - z = 0 \\ x + 2y + 4z + 3t = 0 \\ x - y - 5z - 3t = 0 \end{cases}$$

b/ Find a basis and the dimension of the subspace $W = \{(x, y, z) \mid x - y + z = 0\}$

Q6. Let $u = (1,0,1), v = (2,a,3)$ and $w = (1,4,b)$.

a/ Under which conditions on a and b , $w = (1,4,b)$ is a linear combination of u and v .

b/<u>Without any matrix row-operations</u>, if $a = 2$ and $b = 3$, is $w = (1,4,b)$ a linear combination of u and v

c/ If $W = span\{u, v, w\}$. Under which conditions on a and b , $W = R^3$?

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

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Code 002

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