King Fahd University of Petroleum & Minerals

Department of Mathematics and Statistics

Math 201: Final Exam, Summer 133 (180 minutes)

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Name: **Student ID:** **Section:** **Serial:**

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#	Part I/Answer		#	Part I/Answer
1			8	
2			9	
3			10	
4			11	
5			12	
6			13	
7			14	
	TOTAL			/70

Part II		Grade	
Q1			/10
Q2			/10
Q3			/10
Q4			/10
Q5			/10
Q6			/10
Q7			/10
TOTAL			/70

Part I. Choose the correct answer (no partial credit):

1. The set of points in the space whose coordinates satisfy the two equations

$$x^2 + y^2 = 9 \text{ and } x = z$$

is

(a) an ellipse
(b) a circle
(c) a parabola
(d) a hyperbola
(e) two straight lines

2. The volume of the parallelepiped determined by the vectors $\overrightarrow{u} = \overrightarrow{i} - \overrightarrow{j} + \overrightarrow{k}$, $\overrightarrow{v} = 2\overrightarrow{i} + 3\overrightarrow{j} - \overrightarrow{k}$ and $\overrightarrow{w} = \overrightarrow{j} - \overrightarrow{k}$ is

(a) 2
(b) 4
(c) 6
(d) 9
(e) 10

3. The distance between the planes

$$2x - y + 3z = 4 \text{ and } -4x + 2y - 6z = 6$$

is

(a) $\frac{\sqrt{14}}{2}$

(b) $\frac{7}{2}$

(c) 7

(d) $\frac{\sqrt{7}}{3}$

(e) 0

4. $\lim_{(x,y)\longrightarrow(4,3)} \frac{\sqrt{x}-\sqrt{y+1}}{x-y-1}$

(a) equals $\frac{1}{4}$

(b) equals $\frac{1}{6}$

(c) equals ∞

(d) equals -1

(e) does not exist

5. Let $xe^y + ye^x + e^{x+y} = 2\ln 3 + 3\ln 2 + 6$. The value of $\frac{dy}{dx}$ at $(\ln 2, \ln 3)$ is

   (a) $-\frac{9+\ln 9}{8+\ln 8}$

   (b) $-\frac{9}{8}$

   (c) $-\frac{8+2\ln 3}{7+3\ln 2}$

   (d) $-\frac{\ln 3}{\ln 2}$

   (e) $\frac{1}{2}$

6. Let $\overrightarrow{v}$ be the direction in which the directional derivative of $f(x, y, z) = e^{xyz}\cos(xyz)$ at $P(1, 0, 1)$ attains its maximum value. If $\overrightarrow{u} = 2i + 3j - k$, then $\overrightarrow{u} \bullet \overrightarrow{v} =$

   (a) 3

   (b) 5

   (c) 6

   (d) 0

   (e) -1

7. The set of all values k such that

$$f(x,y) = 3x^2 + 4kxy + 5y^2$$

has a saddle point is:

(a) $(-\infty, -\frac{\sqrt{15}}{2}) \cup (\frac{\sqrt{15}}{2}, \infty)$

(b) $(-\frac{\sqrt{15}}{2}, \frac{\sqrt{15}}{2})$

(c) $(-2, 2)$

(d) $(-\infty, , \infty)$

(e) $\{\ \}$ (the empty set, *i.e.* no such value of k exists)

8. Consider the parametric curve

$$x = t + e^t,\ y = 1 - e^t\ .$$

The value of $\frac{d^2y}{dx^2}$ at $t = 0$ is

(a) $-\frac{1}{8}$

(b) $-\frac{1}{2}$

(c) 1

(d) $\frac{1}{2}$

(e) $\frac{1}{4}$

9. The slope of the tangent line to the polar curve $r = 1+\sin\theta$ at $\theta = 0$ is

   (a) 1
   (b) -1
   (c) 2
   (d) -2
   (e) $\frac{1}{2}$

10. The length of the polar curve $r = e^{\theta}$, $\theta \in [0, \pi]$ is

   (a) $\sqrt{2}(e^{\pi} - 1)$
   (b) e^{π}
   (c) $e^{\pi} - 1$
   (d) $e^{\pi} - 2$
   (e) $\sqrt{2}e^{\pi} - 2$

11. The average value of $f(x,y) = \frac{3xy}{(x^2+1)^2}$ over the region $R = [-1,2] \times [0,1]$ is

(a) $\frac{3}{40}$

(b) $-\frac{3}{40}$

(c) $\frac{3}{20}$

(d) $-\frac{3}{20}$

(e) $\frac{1}{10}$

12. The volume of the solid bounded by the xy-plane, the planes $z = x+1$, $x = 1$, $x = 2$, and the cylinders $y = \pm\frac{1}{x}$ is

(a) $2(\ln 2 + 1)$

(b) $\ln 2 + 1$

(c) $\ln 2 + 2$

(d) $2(\ln 2 + 2)$

(e) $\ln 4 + 1$

13. $\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{dydx}{(1+x^2+y^2)^2} =$

(a) $\frac{\pi}{2}$

(b) π

(c) 2π

(d) π^2

(e) $\frac{\pi}{4}$

14. The volume of the portion of the sphere $x^2 + y^2 + z^2 \leq 1$ that lies outside the cone $x^2 + y^2 = 3z^2$ is

(a) $\frac{2\pi}{3}$

(b) $\frac{\pi}{3}$

(c) 2π

(d) 3π

(e) π

Part I. Solve each of the following 7 questions showing full details.

Q1. Consider the planes

$$x - 2y + 4z = 2 \text{ and } x + y - 2z = 5.$$

(a) Find parametric equations of the line of intersection of the two planes.

(b) Find the equation of the plane through the point (2,3,4) which is perpendicular to the line obtained in (a).

Q2. Find the maximum value which $f(x, y, z) = 3x + 4y + 2z$ attains on the sphere $x^2 + y^2 + z^2 = 25$.

Q3. Prove using the (ϵ, δ)-definition that

$$\lim_{(x,y)\longrightarrow(0,0)} \frac{2xy^2}{x^2+y^2} = 0.$$

Q4. Find the local maxima, local minima and saddle points (if any) of

$$f(x, y) = 2x^3 + 2y^3 - 9x^2 + 3y^2 - 12y.$$

Q5. Convert the integral $I = \int_{-1}^{1} \int_{0}^{\sqrt{1-y^2}} \int_{0}^{x} (x^2+y^2) dz dx dy$ to an integral in cylindrical coordinates, then evaluate it.

Q6. Consider the polar curve

$$r = \frac{1}{2} + \sin\theta.$$

(a) Identify the symmetries in this curve, if any.

(b) Sketch this curve.

(c) State an integral whose value is the length of the portion of the curve for the interval $\theta \in [0, \frac{\pi}{2}]$ (*do not evaluate this integral.*)

Q7. Shade the region inside the circle $r = -2\cos\theta$ and outside the circle $r = 1$, then find its area.

GOOD LUCK